A common (but too restrictive) definition of price discrimination: charging different customers different prices for identical products with the goal of increasing profits. The phenomenon we want to talk about is more general than this, though, because products need not be identical.

Little known fact: price discrimination is illegal in this country (unless justified by cost differences or to match the price of a competitor) – rarely enforced, except occasionally at the wholesale level.

examples:

• just bought tickets at Huntington for Candide:
  – $15 a piece for my children (students have a lower willingness to pay)
  – $80 a piece for my mother- and father-in-law
  – $85 a piece for my husband and me
• Walmart pays less at wholesale level for some of the products it sells than other retailers do
  – bargaining power from size?
• Trader Joes pays less at wholesale level for some of the products it sells than other retailers do
  – bargaining power from willingness to substitute?
• a first-class ticket to China is $9000 but a coach ticket to China is $1000
  – similar products, but not identical but does that arise from cost differences?
• Italian grocery shopping

once you start thinking in terms of price discrimination, you see it everywhere.

traditionally, we have classified price discrimination into three types:

• 1st degree, or perfect
• 2nd degree, or self-selected
• 3rd degree, or based on observables

1st degree

• each consumers’ preferences are completely known
monopolist can make a different take-it-or-leave-it offer to each

consider single consumer’s demand: curve is willingness to pay per visit, so willingness to pay for $Q_0$ units is entire shaded area, or:

$$\int_0^{Q_0} W(t) \, dx$$

e.g., you’re willing to pay 15¢ for the first ounce of coffee, 14¢ for the second ounce, etc. and the marginal cost of an ounce of coffee is 7¢, the optimal thing for a firm to do is to tell you 9 oz of coffee for 99¢

situation is simpler when consumer only buys 0 or 1 unit of product - just charge WTP for everyone who has WTP $> c$

monopolist chooses $Q^*, P^*$ to:

$$\max_{Q,P} (P - cQ) \text{ such that } \int_0^Q W(t) \, dt \geq P \text{ (can do this for each consumer separately)}$$

$P^* = \int_0^{Q^*} W(t) \, dt \text{ (because price is set for each individual)}$

and $W(Q^*) = c \text{ (because you don’t want to sell units where you can only charge } \leq c) $

Notes

solution is to sell same amount as in perfect competition but charge higher price to everyone (but the marginal guy)

can achieve same outcome with two-part tariff $A_i + cQ_i$ where $A_i$ is consumer $i$’s surplus at competitive price

- e.g., Disneyworld, printers, ereaders & ebooks, taxis
- not clear that these two-part tariffs are being used for first degree price discrimination, but they are examples of two-part tariffs

1st degree price discrimination is socially optimal, no DWL (this is not a statement about the distribution of the surplus, only the amount - here all goes to the monopolist)

not so common, really, because monopolist needs so much information about each consumer and must be able to prevent arbitrage
can you think of any market where seller gathers consumer-by-consumer information that could allow them to estimate individual demand curves or WTP?

– Amazon’s disastrous flirtation with individual pricing
– how much information does MIT have about your parents’ WTP?
– do technological advances suggest that 1st-degree price discrimination will become more common?

3rd degree

• monopolist can make different take-it-or-leave-it offers to different observable classes (think about theater tickets)
• 2 populations: \( i = 1, 2 \)
• independent demands \( Q_i(P) \), no arbitrage
• unit demands (for simplicity)

\[
\max P_1, P_2, (P_1 - c)Q_1(P_1) + (P_2 - c)Q_2(P_2)
\]

• 2 focs, one involving \( P_1 \) and the other involving \( P_2 \)
• not surprisingly:

\[
P_i^* - \frac{c}{P_i^*} = -\frac{1}{\epsilon_i}
\]

• if monopolist is not allowed to discriminate:

\[
\max P : (P - c)Q_1(P) + (P - c)Q_2(P)
\]

\[
\text{foc: } \sum_{i=1}^{2} (P_i^* - c)(Q_i'(P^*) + Q_i(P^*)) = 0
\]

Notes

• depending on shape of demand curves and distance between them, monopoly may set \( P^* \in (P_1^*, P_2^*) \) or may set \( P^* = P_2^* \) and only serve one market

• welfare effects are ambiguous – DWL might be mitigated relative to uniform price if output goes up, but there is also a misallocation – some that purchase in low-price group might have lower valuation for good than a non-purchaser in the high price group.

• e.g., Italian grocery shopping (hard to prevent arbitrage) or student/senior ticket discounts (can prevent arbitrage)

• even if this misallocation is inefficient, might be some social or broader economic justification for it. (e.g., “we think it’s sad that parents can’t afford to take their kids to Fenway anymore, so we’ll give family discounts.”)
2nd degree

- this case is more difficult, both for us to analyze and for the seller to pull off, than it is for 1st or 3rd degree because here types or WTP are not observable.
- so, seller must rely on self-selection, but how could that possibly work because consumers will not willingly pay more just because their (unobserved) WTP is higher
- seller must set up a system where high types willingly select into paying a higher price, perhaps because the higher price product is higher quality or higher quantity
  - e.g., first class versus coach class, model/trim/options on new car, premium memberships
- formally, analysis goes exactly the same way regardless of whether we’re talking about higher qualities or quantities:
  
  consumers’ utility is \( u = V(Q, \theta) - T \)
  
  - \( Q = \) quality or quantity
  - \( \theta = \) type
  - \( T = \) total payment

  assume:
  
  \[
  \begin{align*}
  &\frac{\partial V}{\partial Q} > 0, \quad \frac{\partial V}{\partial \theta} > 0, \quad \frac{\partial^2 V}{\partial Q^2} < 0, \quad \frac{\partial^2 V}{\partial \theta \partial Q} > 0 \\
  \end{align*}
  \]

  - utility is increasing in \( Q \) but at a decreasing rate
  - higher \( \theta \) types are willing to pay more for a given \( Q \) and the discrepancy increases as \( Q \) increases
  - high types, higher marginal return to quality

- if \( \theta \) is observable, you can get perfect price discrimination, but you can’t just offer those prices if \( \theta \) is unobservable because high types would pretend they’re low types to get lower prices

- the analysis with a continuum of types is complicated so we’ll simplify by assuming just two types, \( \theta_1, \theta_2, \theta_2 > \theta_1 \)

\[
\begin{align*}
V(\theta, \theta_1) &= V_1(\theta) \quad \text{and} \quad V(\theta, \theta_2) = V_2(\theta) \\
V_1(Q) &= T = c_1 \quad \text{and} \quad V_2(Q) = T = c_2
\end{align*}
\]
• monopolist:
  \[ \max T, Q_1, T_2, Q_2 : T_1 + T_2 - c(Q_1 + Q_2), \text{ such that:} \]
  \[
  \begin{align*}
  V_1(Q_1) - T_1 & \geq 0 & \text{IR}_1 \text{ (individual rationality)} \\
  V_2(Q_2) - T_2 & \geq 0 & \text{IR}_2 \\
  V_1(Q_1) - T_1 & \geq V_2(Q_2) & \text{IC}_1 \text{ (incentive compatibility)} \\
  V_2(Q_2) - T_2 & \geq V_1(Q_1) - T_1 & \text{IC}_2
  \end{align*}
  \]

• becomes a delicate balancing act - make each product attractive enough to buy but not so attractive
  that the other type wants to switch

• what constraints aren’t binding?
  - IC$_2$ + IR$_1$ ⇒ IR$_2$
    \[ V_2(Q_2) - T_2 \geq V_2(Q_1) - T_1 \geq V_1(Q_1) - T_1 \leq 0 \]
  - ignore IC$_1$ for now - we can see that it’s satisfied at optimum

• when maximizing such that IR$_1$ and IC$_2$, note that:
  - $T_1 = V_1(Q_1)$, otherwise raise $T_1$
  - $T_2 = T_1 + V_2(Q_2) - V_2(Q_1)$
    \[ = V_1(Q_1) + V_2(Q_2) - V_2(Q_1), \text{ otherwise raise } T_2 \]

• so we can substitute these equalities in and rewrite as unconstrained maximization:
  \[ \max Q_1, Q_2 : V_1(Q_1) + (V_1(Q_1) + V_2(Q_2) - V_2(Q_1)) - c(Q_1 + Q_2) \]
  \[
  \text{foc: } \frac{\partial V_2}{\partial Q_2} = c \Rightarrow \hat{Q}_2 = Q_2^* \\
  \frac{\partial V_1}{\partial Q_1} = c + \frac{\partial V_2}{\partial Q_1} - \frac{\partial V_1}{\partial Q_1} \Rightarrow \hat{Q}_1 < Q_1^* \\
  \]
  \[ \text{positive number by above assumptions} \]
  - $^* =$ maximizing quality when type observable
  - $^\sim =$ maximizing quality when type unobservable

  to verify IC$_1$, note: $V_1(Q_2) - V_1(Q_1) < V_2(Q_2) - V_2(Q_1) = T_2 - T_1$
  \[ \frac{\partial V_1}{\partial Q_1} > c, V \text{ concave } \Rightarrow \hat{Q}_1 < Q_1^* \]

Observations
• quantity/quality for high type not distorted by price discrimination, but Q distorted down for low type
• reason traveling coach is so crappy is that airlines have to do something to make business travelers not
  want to take it.
• high types receive a surplus, low types do not
• welfare lower than first degree price discrimination, comparison with uniform price ambiguous
Relation to mail-order & Amazon in particular

- noted that catalogs, internet retail sites have lots of information on us
- noted that they also have pretty good tools to prevent arbitrage
- Amazon, at least, has come under intense criticism for attempted 1st and 3rd degree price discrimination
- other mail order houses may still be able to do it  not sure about future
- 2nd degree price discrimination will always be available to retailers who can manage it