1. Let $X_1, \ldots, X_n$ be iid Poisson ($\lambda$) and let $\lambda$ have a Gamma ($\alpha, \beta$) distribution (the conjugate family for Poisson)

$$\pi(\lambda) = \lambda^{\alpha-1} \exp\{-\lambda/\beta\} / \Gamma(\alpha) \beta^\alpha$$

(a) Find the posterior distribution for $\lambda$.

(b) Calculate posterior mean and variance. *Hint: mean of Gamma ($\alpha, \beta$) is $\alpha \beta$; the variance is $\alpha \beta^2$.*

(c) Discuss whether the prior vanishes asymptotically.

(d) Assume that $\alpha$ is an integer. Show that the posterior for $2(n\beta+1)\lambda$ given $X$ is $\chi^2(2(\alpha + \Sigma X_i))$.

(e) Using result of (d), suggest a 95%-credible interval for $\lambda$.

2. Suppose that conditional on $\tau$ a random variable $X$ has normal distribution with mean zero and variance $\frac{1}{\tau}$. The prior for $\tau$ is Gamma ($\alpha, \beta$).

(a) Find the posterior for $\tau$.

(b) Compare the prior mean for $\tau$ and the posterior mean.

3. Let $X$ be a random variable with exponential distribution

$$f(x \mid \beta) = \frac{1}{\beta} e^{-x/\beta} ; x > 0, \beta > 0.$$ 

One wants to test $H_0 : \beta = \beta_0$ against $H_a : \beta \neq \beta_0$.

(a) Suggest a 5% level test.

(b) Draw the power function.

(c) Provide the formula for the $p$-value.
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