1. Answer two questions out of the three.
   14.382 Econometrics I
   Final Examination
   Spring, May 2004
   (Professor Jerry Hausman)

INSTRUCTIONS: (2 hour final exam)

2. Let \( y = \beta_1 x_1 + \epsilon \) where \( x_1 = x_1^* + \nu \)
   where \( E(\nu) = 0, E(x_1^*, \nu) = 0, E(\epsilon, x_1^*) = 0 \).
   (i) Suppose you do least squares. Derive the plim of \( \hat{\beta} \)
       and demonstrate "attenuation bias." ("iron law" of econometrics)
   (ii) Suppose you have an instrument \( z \). What properties must \( z \)
        have to be a valid instrument? Give a proof that the IV estimation is consistent.
   (iii) Suppose the specification is \( y_1 = \beta_1 x_1 + \beta_2 x_2 + \epsilon \), where \( Cov(x_1, x_2) \neq 0 \) and
        \( E(x_2, \epsilon) = 0 \). Determine the large sample bias in \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \).
        (Hint: partial out \( x_2 \)).
   (iv) Does the "iron law" of econometrics hold for \( \hat{\beta}_1 \) (downward bias in magnitude). Does the presence of \( x_2 \)
        lead to less or more large sample bias in \( \hat{\beta}_1 \)?

3. You have a panel data model:

\[
y_{it} = X_{it}\bar{\beta} + Z_{it}\gamma_i + \alpha_i + \eta_{it}
\]

\[
i = 1, \ldots, N; t = 1, \ldots, T
\]

Where \( N \) is large and \( T \) is small.
   (i) How should you test \( H_0: E(\alpha_i | X_{it}, Z_i) = 0 \)?
   (ii) You run fixed effects estimation and do an F test that \( H_0: \hat{\alpha}_1 = \hat{\alpha}_2 = \ldots = \hat{\alpha}_N = 0 \)
       Specify the test. What should you conclude about your estimates of \( \beta \) and \( \gamma \) if you reject \( H_0 \)?
   (iii) Suppose you think you may have errors in variables (EIV) in one of the \( X_{it} \)'s:
       \( X_{it} = X_{it}^* + \nu_{it} \), where \( E(\nu_{it}) = 0, E(x_{it}^*, \nu_{it}) = 0 \) for \( t \neq \tau \) and \( E(X_{it}^*, \nu_{it}) = 0 \). What effect could EIV have on your fixed effects estimates and your test of \( E(\alpha_i | X_{it}, Z_i) = 0 \)?
   (iv) How could you test if you do have an EIV problem? Can you give a consistent estimator if you do have an EIV problem?

4. You have a Tobit Model:

\[
y_i^* = X_i\beta + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0, \sigma^2), \quad i = 1, \ldots, N.
\]

And

\[
y_i = y_i^* \quad \text{if } y_i^* < S_i
\]

\[
y_i = S_i \quad \text{if } y_i^* \geq S_i
\]

(i) Write down the likelihood function (LF) where \( S_i = S_j \) for all \( i, j \). Then generalize the
    LF where \( S_i \neq S_j \)
   (ii) Demonstrate "Fisher Consistency" for the situations where \( S_i \neq S_j \).
   (iii) Suppose you observe the \( S_i \)'s with error: \( S_i = S_i^* + \nu_i \), where the \( S_i^* \) are not observed
       and \( E(S_i^*, \nu_i) = 0, E(\nu_i) = 0 \), and \( E(S_i) = 0 \). What is the effect on the ML estimates?
   (iv) Suppose you decide to test the model specification. You do a probit model for \( y_i < S_i \)
       or \( y_i = S_i \). You compare these results to the ML Tobit Model estimate. Give a test and
determine its properties.