14.385 Problems Set 3A  
Fall 2007

These are two problems that will be included in problem set 3. There will be an additional one or two problems.

1. Consider a linear panel data model

\[ Y_{it} = X'_{it} \beta + \alpha_i + \eta_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T. \]

Suppose that conditional on \( X_{i1}, ..., X_{iT} \) and \( \alpha_i \), the vector \((\eta_{i1}, ..., \eta_{iT})'\) is distributed as \( N(0, \sigma^2 I_T) \).

a) What is the conditional density \( f(Y_{i1}, ..., Y_{iT} \mid X_{i1}, ..., X_{iT}, \alpha_i; \beta, \sigma^2) \)?

b) Calculate the plim of the estimator of \( \sigma^2 \) from maximizing the conditional log-likelihood over \( \sigma^2, \beta, \) and \( \alpha_1, ..., \alpha_n \).

c) What is the conditional density \( f(Y_{i1}, ..., Y_{iT} \mid X_{i1}, ..., X_{iT}, \alpha_i; \sum_{t=1}^T Y_{it}; \beta, \sigma^2) \)?

d) For the conditional density in c), find the conditional MLE for \( \beta \) and \( \sigma^2 \), and show that the estimator of \( \sigma^2 \) is consistent.

2. Suppose that \( Y_{it} \) is count data with \( Y_{it} \in \{0, 1, 2, ...\} \), and that \( E[Y_{it} \mid X_{it}, \alpha_i] = \exp(X'_{it} \beta_0 + \alpha_i) \).

a. Consider an estimator of \( \beta \) that is obtained as

\[ \tilde{\beta} = \arg \min_{\beta, \alpha_1, ..., \alpha_n} \sum_{i,t} \{Y_{it} - \exp(X'_{it} \beta_0 + \alpha_i)\}^2. \]

Is this estimator consistent? Why or why not.

b. Consider the function \( \rho_{it}(\beta) = Y_{it} \exp(X'_{it, t-1} \beta) - Y_{i, t-1} \exp(X'_{it} \beta) \). Show that \( E[\rho_{it}(\beta_0)|X_{i}] = 0 \).

c. How could you use the result from b) to form an estimator of \( \beta \)?

d. (extra credit) Consider the conditional maximum likelihood estimator obtained when conditional on \( X_{it} \) and \( \alpha_i \), the variables \( Y_{i1}, ..., Y_{iT} \) are independent Poisson with \( E[Y_{it} \mid X_{it}, \alpha_i] = \exp(X'_{it} \beta_0 + \alpha_i) \) and we condition on \( S_i = \sum_{t=1}^T Y_{it} \) is consistent if only the conditional mean assumption is satisfied (i.e. they need not be Poisson). Hint: A sum of independent Poissons is Poisson.