PASTURE 1: OVERVIEW
Taxonomy of resources.
Question: What are examples of depletable, renewable, and expendable resources?

<table>
<thead>
<tr>
<th>Depletable</th>
<th>Renewable</th>
<th>Expendable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil</td>
<td>Forests</td>
<td>Wind</td>
</tr>
<tr>
<td>Natural gas</td>
<td>Fish</td>
<td>Grains</td>
</tr>
<tr>
<td>Groundwater (?)</td>
<td>Solar radiation</td>
<td></td>
</tr>
</tbody>
</table>

Point out that the categorization depends really on how you would sensibly model them. E.g. Oil does replenish, but very slowly.

PASTURE 2: TWO PERIOD MODEL, EXOGENOUS PRICE
Model 1:
Setup:
Two periods.
Exogenous prices, no uncertainty
Maximum total availability:
\[ Q_f = E_0 + E_1 \]
Cost = \( C(E) \)

Firm maximizes profits:
\[ \pi = p_0 E_0 - C(E_0) + \delta [p_1 (Q_f - E_0)] \quad \text{s.t.} \quad Q_f = E_0 + E_2 \]

Question: What conditions do we need on cost function to get an interior solution?
\[ dC/dE > 0 \]
\[ d^2 C/dE^2 > 0 \]
Example cost function: \( C(E) = \gamma E + \theta E^2 \)

Sub in resource constraint:
\[ \pi = p_0 E_0 - C(E_0) + \delta [p_1 (Q_f - E_0) - C(Q_f - E_0)] \]
Take derivative:
\[ p_0 - C'(E_0) = \delta [p_1 - C'(E_2)] \]

Question: What is the verbal interpretation of this?
Answers:
- We have a fixed amount of resource. Equalize the NPV of the value of exploiting the resource now versus later.
- MB=MD interpretation (pollution analogy)
- MS=MC interpretation (pollution abatement analogy)
- Supply and demand analogy. Demand in period 1 is in essence the supply in period 0.

- Draw figure

Question: What if I relax the resource constraint, i.e. make Q bigger? 
Answer: Then we have a standard non-scarce good, and production in the two periods is independent.

Model 2: Hotelling Costs

“Hotelling costs“: Constant marginal costs in each period c,
Corner solution: Extract all in period 0 if:
\( (p_0 - c) > \delta (p_1 - c) \)

Result: Unless prices fall, we extract everything now.

Question: What does this mean for what prices have to be?
Push question: what if all firms homogeneous but there is non-zero demand elasticity? Then need to have some extraction in P0 and some in P1 in order for prices not to be zero or infinite.

Things we know:
- Profits grow but PV of profits shrinks?
- Resource will be totally depleted?

PASTURE 3: LAGRANGIAN

Question: How many people know what the LaGrangian is?

Model 3: Two Period LaGrangian

Same as Model 1 above
\[
\begin{align*}
\pi &= p_0 E_0 - C(E_0) + \delta (p_1 E_1 - C(E_1)) \\
L &= p_0 E_0 - C(E_0) + \delta (p_1 E_1 - C(E_1)) + \lambda (Q_f - (E_0 + E_1))
\end{align*}
\]

\[
\begin{align*}
dL/dE_0 &= p_0 - C'(E_0) \lambda = 0 \\
dL/dE_0 &= \delta [p_1 - C'(E_1)] \lambda = 0
\end{align*}
\]

Question: What is \( \lambda \) mathematically?
\( \lambda = \delta [p_1 - C'(E_1)] \)
\[ \lambda = \delta \left[ p_0 - C'(E_0) \right] \]

Question: What is \( \lambda \) in words?
Answer: Opportunity cost of extracting the marginal unit, in present value
Also known as the scarcity rent, shadow value, shadow price, shadow cost.
**** Perhaps the most important concept in this class.

Question: What if the constraint didn’t bind? What’s the value of \( \lambda \)?
\( \lambda = 0 \)
Notice that this now looks like a standard (abundant, non-scarce) resource.

- PowerPoint vignette: Betting the Planet

**Model 4: Multi-Period LaGrangian**

\[ \sum \delta^t \left[ p_t E_t - C(E_t) \right] + \lambda \left[ Q - \sum E_t \right] \]
General solution:
\[ \delta^t \left[ p_t - C'(E_t) \right] = \lambda \]
\[ p_t - C'(E_t) = \lambda / \delta^t \]
“Markup rises at the interest rate.”
Intuition: Storable good: I can store money or store the good between periods. I should be indifferent between storing those two things

**Hotelling’s Rule:**
Assuming Hotelling Costs:
\[ p_t = c_t + \lambda / \delta^t \]
If marginal cost is constant, price rises at the interest rate

Question: What’s the present value of the resource?
\[ \sum \delta^t \left[ p_t E_t - C(E_t) \right] = \lambda Q \]
Intuition?

- PowerPoint Vignette: Are we running out of natural gas?

**PASTURE 4: DEMAND SIDE, PRICES, AND EQUILIBRIUM**

**Setup:**
**Supply Side**
Supply has Hotelling costs. Firms are price takers (competitive supply)
Total resource availability: \( Q_T = F Q_r \)
\( F \)=number of firms

**Equilibrium prices:**
For an interior solution (with production in each period), firms must be indifferent, so the Hotelling Rule has to hold:
\[ \delta^i \left[ p_t - c_t \right] = \lambda \]
\[ p_t = c_t + \frac{\lambda}{\delta^i} \]

**Demand Side**
Demand: \( E_t = A - Bp_t \)
\[ E_0 = \left[ A_0 - c_0 - \lambda \right] / B \]
\[ \lambda = \delta \left( p_1 - c_1 \right) \]
\[ \lambda = A - BE_0 - c_0 \]

**Equilibrium Quantities:**
\[ E_0 = \left[ A_0 - A_1 - \delta A_1 - c_0 + \delta c_1 + \delta Q_t B \right] / \left[ B \left( 1 + \delta \right) \right] \]

**Comparative Static:** Technological change
How to model?
Cost reductions.
What is your intuition? Naïve intuition: Cost reductions should reduce price, increase quantity.

\( E_0 \) and \( E_1 \): sum is fixed, so quantity can’t increase.
\( E_0 \) and \( E_1 \): really depend on the relative cost reductions between the two periods: it still matters when it is more profitable to extract.
If \( E_0 \) and \( E_1 \) stay the same, then price remains the same, and consumers do not gain at all from cost reduction. Firms capture it all through increase in scarcity rents.

**Comparative Static:** Exploration.
Question: How to model this?
Answer: An increase in \( Q_t \)
This increases \( E \) in both periods and decreases price.

**Comparative Static:** Demand Growth
Unanticipated vs. anticipated:
If unanticipated, Hotelling rule no longer holds
If anticipated, Hotelling rule holds and \( E_0 \) is lower while \( E_1 \) is higher than without demand growth.

**Comparative Static:** Discount Factor
What if the discount factor decreases (interest rate increases)?
Firms extract more now, less later.
If firms’ discount rate different from social discount rate, we are off the optimum.

**Social Optimum**
Question: Is this the social optimum?
Answer: Yes. There are no market failures.

Dynamic efficiency. Maximize social surplus over periods.
Two period model – graphical.
Question: is it “fair” to consume more now? Yes! Intertemporal pareto efficiency.
This is not “sustainable,” however: we are consuming

**PASTURE 6: DEVIATIONS FROM SOCIAL OPTIMUM**

**Model 1: OPEC/Cartels**

\[
\max_{E_t} \delta^t \left[ p_t(E_t)E_t - c_tE_t \right] + \lambda \left[ Q_t - \Sigma_t E_t \right]
\]

MR-MC-\(\lambda=0\)
MR = MC+\(\lambda\)
Looks like the normal monopoly condition, except now there is a shadow price added to marginal cost.
\(A_0-2BE_0 - c_0 = \delta[A_1-2BE_1 - c_1]\)
\(=\delta[A_1-2B(Q_t-E_0) - c_1]\)
\(E_0= [A_0-\delta A_1- c_0+ \delta c_1 + 2B\delta Q_t] / [2B(1+\delta)]\)

**Model 2: Externalities**

Externalities from national security or climate change

**Other Extensions Possible:**
Substitutes elastically available at a given marginal cost – substitution from oil to solar, for example.
Stock effects on cost: solve sufficient conditions
Numerical solutions