1 The consumer’s budget constraint and his/her assets

- One can imagine the models in two ways:

1. First, with positive supply of loans:
   - There is an innovation sector, which borrows to finance its input costs (either in units of labor or final goods), and it pays interest on its loans
   - Loans are held by agents

2. Second, with zero supply of loans:
   - In this case, the research part of the labor force $L_R$ is paid not in terms of current wages, but in terms of future profits (think of this as an “equity” stake in a firm in the innovation sector)

2 Example: Knowledge spillovers model

- Reminder: Equilibrium quantities and prices were

$$Y = \frac{1}{1-\beta}NL_E$$
$$X = (1-\beta)NL_E$$
$$w = \frac{\beta}{1-\beta}N = \eta NV$$
$$\pi = \beta L_E$$  \hspace{1cm} (1)

- Let us first determine $C$ from the resource constraint of the economy,

$$C + X = Y$$

so

$$C = \frac{1 - (1-\beta)^2}{1-\beta}NL_E = \frac{2\beta - \beta^2}{1-\beta}NL_E$$

- We now write down the specific budget constraints corresponding to the two interpretations above

2.1 First interpretation: Positive supply of loans

- We can express the consumer’s budget constraint as

$$C + \dot{A} \leq rA + wL$$
where \( A \equiv NV \) is the total value of outstanding loans. Notice:

\[
rA = N\pi + NV = N\pi + \dot{A} - \dot{NV}
\]
giving

\[
C = rA - \dot{A} + wL = N\pi + wL - NV = N\pi + wL_E,
\]
where in the last equality we used that

\[
\dot{N} = \eta NL_R = \frac{wL_R}{V}
\]
using the expression for the wage (1).

- This yields the exact same expression for consumption,

\[
C = N\pi + wL_E = \beta NL_E + \frac{\beta}{1-\beta}NL_E = \frac{2\beta-\beta^2}{1-\beta}NL_E.
\]

### 2.2 Second interpretation: Zero supply of loans and equity stakes

- In this case, consumption is given by

\[
C = \overbrace{wL_E}^{\text{current labor income}} + \overbrace{N\pi}^{\text{dividends from equity holdings}}
\]

and this is again the same as before,

\[
C = N\pi + wL_E = \beta NL_E + \frac{\beta}{1-\beta}NL_E = \frac{2\beta-\beta^2}{1-\beta}NL_E.
\]