14.452 Review session

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Logistics

- Exam next Monday
- Greg will proctor
- Open book & lecture notes
- 3-4 short questions, 1-2 long questions
Determinants of growth

- Definition

\[ Y = F(A, K, L, H) \]

where

- \( A \) = technology
- \( K \) = physical capital
- \( L \) = labor force
- \( H \) = human capital / education

- Only **proximate causes, not fundamental**
  - such as geography, luck, institutions, preferences
  - Acemoglu Naidu Restrepo Robinson (2014): Democracy causes \( \approx 1\% \) higher GDP growth
Why write a model of growth?

• For each proximate cause $X$, want guidance on: (among others)
  • How do fundamental causes affect the growth of $X$?
  • Under what conditions can there be sustained growth in $X$?
  • What kind of policies can help accumulate more $X$?
  • What kind of policies can increase welfare? (if at all?)
  • How can we measure contribution of growth in $X$ empirically?

• These Qs require a model with endogenous accumulation of $X$
  • will do this for $A, K$. $H$ similar to $K$
Common theme

- In background: \( \exists \) “accumulation technology” of \( X \)
  - concave \( \Rightarrow \) exogenous growth
  - linear \( \Rightarrow \) endogenous growth
An aside on TVCs

- TVC: part of **sufficient conditions** for optimum in any **infinite horizon optimal control problem**
  - e.g. a representative household’s problem, or a planning problem

- When there is a some lower bound on wealth, it is

\[
\lim_{t \to \infty} e^{-(\rho-n)t} u'(c_t) \text{wealth}_t = 0
\]

so we can write

\[
\lim_{t \to \infty} e^{-rt} \text{TotalWealth}_t = 0
\]

where TotalWealth is the whole current generation’s wealth

- In pretty much any model, TotalWealth grows at rate \( gY \), so along BGP this means

\[
r > g \text{TotalWealth} = gY
\]
Outline

1. **Solow model: \( K \)**
   - Uzawa’s theorem
   - Solow models
   - Data

2. **NGM and OLG: still \( K \)**
   - NGM
   - OLG & dynamic inefficiency

3. **Neoclassical endogenous growth: still \( K \)**

4. **Endogenous technology: \( A \)**

5. **World technology growth: \( A \)**

6. **DTC: What kind of \( A \)?**
Section 1

Solow model: $K$
Subsection 1

Uzawa’s theorem
How should technology affect production?

- Could be Hicks, Solow, Harrod neutral
- Uzawa: If \( Y = \tilde{F}(K, L, t) \) and
  - capital accumulates as \( \dot{K} = Y - C - \delta K \)
  - \( K, Y, C \) grow exponentially
- Then:
  - \( g_K = g_Y = g_C \)
  - **can always write it as Harrod neutral**, \( Y = F(K, A(t)L) \) for some \( A(t) \), \( g_A = g_Y - n \)
  - if \( R = \tilde{F}_K = \text{const} \Rightarrow R = F_K = \tilde{F}_K \)
Subsection 2

Solow models
Solow model: concave accumulation

- Using Uzawa $\Rightarrow$ focus on $Y = F(K, AL)$
- Constant savings rate $s$
- Capital accumulation

$$\dot{K} = sF(K, AL) - \delta K$$

- $A$ exogenous, $F$ CRS, with Inada conditions
- Solve? $\rightarrow$ Recitation #2
Results: exogenous growth

- Define $k \equiv K/(AL)$ (more generally $k \equiv e^{-gt}K$)
- Unique positive steady state $k^*$, globally stable

$$\frac{f(k^*)}{k^*} = \frac{\delta + n + g}{s}$$

- Exogenous growth, $\dot{Y}/Y = n + g$
- If you can pick $s$, i.e. $k^* = k^*(s)$, consumption largest if $k^*(s) = k^*_{gold}$ (golden rule)

$$f'(k^*_{gold}) = \delta + n + g$$

- $k^* > k^*_{gold}$: have "dynamic inefficiency" (but not well defined here)
AK version: sustained growth

- Fix $A$.
- $F = AK \Rightarrow$
  \[ \dot{K} = sAK - \delta K \]
  \[ g_K = sA - \delta \]
- No transitional dynamics
Subsection 3

Data
How much does each proximate cause account for growth?

- Within countries: **Growth accounting**

\[
g_Y = sKg_K + sLg_L + \varphi \text{ effect of } A
\]

- OECD countries: 40-50% capital, 30-50% TFP
- LDCs: less TFP, more labor
- mismeasurement issues from capital prices & human capital
How much does each proximate cause account for cross-country GDP differences?

- **Across countries:** Development accounting
- **Idea:** Make functional form assumption for $Y$ and compare across countries, e.g.
  \[
  \frac{Y}{L} = A \left( \frac{K}{L} \right)^{\alpha} \left( \frac{H}{L} \right)^{\beta}
  \]
- **Two approaches:**
  1. Assume $A_j$ exogenous $\Rightarrow$ figure out $\alpha, \beta$ & $R^2$
  2. Pick value for $\alpha, \beta$ $\Rightarrow$ Recover $A_j$'s
1) Mankiw Romer Weil

- Assume Solow-type accumulation of $K$ and $H$ → \textbf{evaluate at steady state}

$$\log y_j^* = gt + \frac{\alpha}{1 - \alpha - \beta} \log \frac{s_{k,j}}{n_j + g + \delta_k}$$

$$+ \frac{\beta}{1 - \alpha - \beta} \log \frac{s_{h,j}}{n_j + g + \delta_h} + \log A_j$$

- Large $R^2$ around 70%, $\alpha, \beta \approx 0.30$

- \textbf{But:}

- Strong assumption that $\log A_j$ is uncorrelated with $s_{k,j}, s_{h,j}$
  - biases $\alpha, \beta, R^2$ upwards

- Huge value of $\beta$ relative to Mincerian estimates
2) Hall Jones 1999

- Construct $H$ from Mincerian regression
- Recover
  \[
  \frac{A_j}{A_{US}} = \left( \frac{Y_j}{Y_{US}} \right)^{3/2} \left( \frac{K_{US}}{K_j} \right)^{1/2} \left( \frac{H_{US}}{H_J} \right)
  \]
- Find larger role for technology
- Assumptions
  - no human capital externalities + other assumptions to construct $K, H$
  - Cobb-Douglas $Y$ with same $\alpha$! (→ can be somewhat more flexible)
Section 2

NGM and OLG: still $K$
Subsection 1

NGM
Baseline NGM

- Endogenize savings rate: Representative household solving

$$\max_{c,k} \int_0^\infty e^{-(\rho-n)t} u(c_t) dt$$

- Assume $\rho > n$, $u(c) = \frac{c^{1-\theta}}{1-\theta}$. For now: $A = 1$.
- Equilibrium efficient (single agent) $\Rightarrow$ Planner

$$\max_{c,k} \int_0^\infty e^{-(\rho-n)t} u(c_t) dt$$

$$c_t + \dot{k}_t = f(k) - (\delta + n)k$$

$k_0$ given
NGM FOCs

- Euler (always holds for \textit{per capita} $c$)

\[
\frac{\dot{c}}{c} = \frac{1}{\theta} \left( f'(k) - \delta - \rho \right)
\]

- TVC

\[
\lim_{t \to \infty} e^{-(\rho-n)t} u'(c_t) k_t = 0
\]

- Illustrate dynamics in \textit{phase diagram}. TVC pins down a single stable arm!

- Can do comparative dynamics ...

- With growth: Use $c/A$ and $k/A$
Subsection 2

OLG & dynamic inefficiency
The problem with infinite households

- With $\infty$ households, planner is allowed to redistribute along an infinite chain of households.

- Can violate FWT if value of endowments is infinite $\rightarrow$ dynamic inefficiency.

- Here: only canonical OLG model with
  - $L = \text{const}$
  - Cobb-Douglas technology $f(k) = k^\alpha$
  - log utility
  - $\delta = 1$
Canonical OLG model

- Generation $t$ solves

$$\max \log c_1(t) + \beta \log c_2(t)$$

$$c_1(t) + k(t) \leq w(t)$$

$$c_2(t) \leq R(t + 1)k(t)$$

Giving

$$k(t) = \frac{\beta}{1 + \beta} w(t) = \frac{\beta}{1 + \beta} (1 - \alpha)k(t)^\alpha$$

- Unique positive steady state $k^*$, globally stable
Dynamic inefficiency

- **But:** possibly $k^* > k^*_{gold}$, i.e. $R^* < 1$: **dynamic inefficiency**

- Can be cured by
  - redistribution from young to old (unfunded social security)
  - less saving
  - government debt
  - money
Section 3

Neoclassical endogenous growth: still $K$
Neoclassical AK model

• Except for the Solow AK economy: No **endogenous** growth model so far! Here: NGM version of AK...

• Assume \( f(k) = Ak \) ⇒

\[
\frac{\dot{c}}{c} = \frac{1}{\theta} (A - \delta - \rho)
\]

\[
\dot{k}_t = Ak - (\delta + n)k - c
\]

• Hence \( g_c = \frac{1}{\theta} (A - \delta - \rho) \), \( r = A - \delta \)

• Need:

\[
r > g_Y = g_C = g_c + n
\]

• Here: Tax changes affect growth rates!
Rebelo AK

- Same AK structure now produces capital, using capital as input
- Final output is consumed \( C = BK_C^\alpha L_C^{1-\alpha} \), relative price of capital goes to zero
- Easiest way to analyze: Planning problem!
Romer 1986: Growth with externalities

- Assume $Y = F(K, AL)$ with $A = BK$ uninternalized “learning by doing”
- Then:
  $$R = F_K(K, AL) = F_K(1, BL) = const$$
  so from Euler we get $g_C = \frac{1}{\theta} (R - \delta - \rho)$
- TVC requires
  $$r > g_Y = g_C$$
- Not Pareto optimal due to externalities!
Section 4

Endogenous technology: A
Endogenous technology models

- Discussed the mechanics in Recitation #4 at length. Here: Overview

- 3 models of endogenous $A$:
  - Lab Equipment, Knowledge Spillovers: expanding varieties $N$
  - Schumpeterian: quality $Q$

- Key: Technology is excludable, even if non-rival
  - hence inventors can earn monopoly rents

- Abstract from $K$
Lab Equipment (Romer 1990)

- Innovation possibilities frontier: $\dot{N} = \eta Z$
- Find BGP with $r = \eta \beta L$ and $g_C = \frac{1}{\theta} (\eta \beta L - \rho)$
- Two types of externalities
  - “new good” externalities
  - monopoly distortion / aggregate demand externalities

$\Rightarrow$ social planner values varieties more & prefers higher growth!

- Implement using two instruments:
  - subsidies to research
  - subsidies to intermediate good inputs

- More competition lowers growth! (but raises current output)
Knowledge spillovers

- Innovation possibilities frontier: \( \dot{N} = \eta N L_R \)
- Find BGP with \( r = (1 - \beta) (\eta L - g) \)
- New externality: Spillovers \( \rightarrow \) even stronger reason for planner to boost growth!
Scale effects

- These models have **scale effects**
- Higher $L \Rightarrow$ higher growth rate
- **Problematic** because
  - $L$ grows in practice
  - higher $L \not\Rightarrow$ higher growth
- Variant: $\dot{N} = \eta N^\phi L_R$, $\phi < 1$ but population growth
- akin to “concave” technology, hence exogenous growth $g_Y = \frac{n}{1-\phi} + n$
Schumpeterian model

- Quality improvements, rather than more gadgets
- Creative destruction
- Find $r = \eta \lambda \beta L - \frac{g}{\lambda - 1}$
- New **business stealing** externality
- Planner does not necessarily want to boost growth!
Section 5

World technology growth: A
Model with technology spillovers

- Lab Equipment model in each country, “anchored” to world technology $N_t = e^{gt} N_0$

$$\dot{N}_j = \eta_j \left( \frac{N}{N_j} \right)^\phi Z_j$$

where $\phi > 0$. At BGP:

$$g_{N_j} = g$$

$$\frac{N_j}{N} = \left( \frac{\eta_j \beta L_j}{\zeta_j r^*} \right)^{1/\phi}$$

- If $N = \frac{1}{J} \sum N_j \Rightarrow$

$$g = \frac{1}{\theta} \left( \frac{1}{J} \sum \left( \frac{\eta_j \beta L_j}{\zeta_j} \right)^{1/\phi} \right)^\phi$$
Remarks

- \( g \) taken as given by each country, but endogenously determined by the countries

- Instead of modelling technology spillovers, terms of trade effects can also synchronize growth rates along the world
  - opposite also interesting: trade causing asymmetric growth rates (e.g. "infant industries")
Section 6

DTC: What kind of $A$?
Why DTC?

- Technology often **directed at certain factors** (e.g. skill biased techn change)
- E.g.
  \[ Y = F(A_L L, A_H H) \]
- What determines profitability of that? e.g.
  \[ \frac{\partial Y}{\partial A_H} = F_A H H \times H \]
  price per efficiency unit market size
- Let \( s_H \) be share of income going to \( A_H H \)
- Then:
  \[ \frac{\partial Y}{\partial A_H} = \frac{Y}{A_H} s_H \]
Relative profitability

- This gives a measure for relative profitability:
  \[
  \frac{\partial Y}{\partial A_H} = \left( \frac{A_H}{A_L} \right)^{-1} \frac{s_H}{s_L}
  \]
- with CES with ES \( \epsilon \): \( s_H / s_L \) depends on \( A_H H / A_L L \)
  - increasing if \( \epsilon > 1 \)
  - decreasing if \( \epsilon < 1 \)
Equilibrium bias

- **Weak equilibrium bias:** Increase in $H/L$ ⇒
  - $A_H/A_L$ increases if $\epsilon > 1$
  - $A_H/A_L$ decreases if $\epsilon < 1$
- Both times: technology response biased towards $H/L$!
- **Strong equilibrium bias:** Increase in $H/L$ ⇒ relative wage $w_H/w_L$ increases
- Upward sloping demand curve
Endogenous DTC model

- Benefit of innovating in sector $H$

\[ V_H = \frac{\beta p_H^{1/\beta}}{r^*} H \]

\[ \frac{V_H}{V_L} = \text{const} \times \left( \frac{N_H}{N_L} \right)^{-1} \left( \frac{N_H H}{N_L L} \right)^{(\sigma-1)/\sigma} \sim s_H / s_L \]

- BGP: $V_H / V_L = \eta_L / \eta_H \Rightarrow$

\[ \frac{N_H}{N_L} = \text{const} \times \left( \frac{H}{L} \right)^{\sigma-1} \]