Problem Set 3
Due Date: April 30th
We encourage you to work together, as long as you write your own solutions.

Attacks of laziness and fluctuations (Waiver 2005)
Assume utility to be separable in leisure and consumption in an otherwise conventional competitive RBC model.
1. Describe the effects of a transitory shock to the utility of leisure (making leisure less enjoyable) on output, employment, consumption, capital accumulation, real wages and interest rates.
2. "We can think of fluctuations as being primarily driven by fluctuations in the utility of leisure." Discuss, in the light of the stylized facts on fluctuations.
3. Would the model be useful to look at seasonal rather than cyclical fluctuations?

Intertemporal Labor Supply
Consider the following problem. The consumer problem is:

\[ \text{Max} \quad E_0 \left( \sum_{t=0}^{T} \beta^t \left( \chi_1 C_t^{\sigma_1} + \chi_2 N_t^{\sigma_2} \right) \right), \]

s.t.
\[ A_{t+1} = R(A_t + N_t W_t - C_t) \]

where \( C \) is consumption, \( N \) is labor supply, \( A_0 \) is initial wealth, \( R = 1 + r \), and the greek letters are parameters. Only \( W \) is stochastic.
1. Derive and interpret the first-order conditions for \( C_t \) and \( N_t \). What are the intertemporal and intratemporal optimal conditions?
2. What is the link between \( C_t \) and \( N_t \) in this model? What assumption(s) is (are) producing this result?
3. How can you analyze changes to wages that do not affect the expected wealth of the consumers? (Hint: what is \( \lambda \)--the Lagrange multiplier--in this model?). Let's define the wealth-constant elasticity of labor supply as \( \eta = \frac{\partial \ln N_t}{\partial \ln W_t} \) when wealth is constant. What is \( \eta \) in this model? Is it positive or negative? Why?
4. Take the FOCs and discuss the effect of the following situations on \( \{C_t\} \) and \( \{N_t\} \).
   - Cross-sectional differences associated with permanent differences in human capital (assume \( W \) varies in the cross-section).
   - Life-cycle changes in wages (i.e. if Mincerian equations are correct, wages follow an inverted-U behavior over the life-cycle).

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• Unexpected temporary shock to wages.
• Unexpected permanent shock to wages.

**Marginal and Average Q**

The problem is that \( q \) is not observable (explain why). However, we can observe Tobin’s \( Q \), or average \( Q \). It is defined as the market value of the company (debt + equity) divided by the book value of its capital stock. In fact, because of the timing convention of the model, we need to define it as the market value net of the current period cash flows. Thus, in discrete time, this expression becomes:

\[
Q_t = \frac{V_t}{K_{t+1}}
\]

In the lecture notes it is shown, in discrete time, that under constant returns to scale, competitive goods markets, a correct measure of capital, and correct valuation of firms by financial markets, average \( Q \) is equal to marginal \( q \).

1. Prove the equivalence of marginal and average \( q \) in the model covered in lecture (lecture 4). (Note: you can use the continuous time version of the model if you prefer.)
2. Discuss for each of the assumptions, when they are used in the proof and why are they essential.
3. Give an example for each of the assumptions of a set up in which one of them is not satisfied and discuss in which sense \( q \) and \( Q \) differ.
4. How does the difference between \( q \) and \( Q \) evolve over the cycle in each of the cases?
5. Today, in Europe, stock price indices are high, and so is Tobin’s \( q \). Despite this, investment spending in Europe is low. What could explain this combination? (Don’t worry if you do not know the answer. This is a hot topic in Europe, and there are many hypotheses... You are just asked to speculate)

**Investment, Tobin’s \( q \), and the Stock Market**

Suppose that all assumptions underlying the equality of marginal and average \( q \) are satisfied, except for market mispricing. Suppose the market’s assessment of average \( q \) is higher than the firm’s assessment of marginal \( q \). What should the firm do? Ignore the market, and invest until marginal \( q = 1 \). Or invest, issue shares until average \( q = 1 \). (There is no right or wrong answer)