14.462
Problem Set 2

Problem 1

In this problem you will replicate Figures on pages 12 and 14 of the lecture notes (demand shocks, part I). Consider a stochastic growth model with preferences and technology given by

\[ U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\eta} N_t^{1+\eta} \]
\[ A_t F(K_{t-1}, N_t) = A_t K_{t-1}^\alpha N_t^{1-\alpha}. \]

The process for \( A_t \) is as follows

\[ A_t = e^{\alpha t}, \]
\[ a_t = \rho a_{t-1} + \epsilon_t. \]

Use parameters

\[ \beta = 0.99, \quad \delta = 0.025, \]
\[ \eta = 1, \quad \sigma = 1, \]
\[ \alpha = 0.36, \quad \rho = 0.95. \]

You can use the Matlab package Dynare (http://www.cepmap.cnrs.fr/dynare/).

(i) Setup the planner problem and derive the first order conditions.
(ii) Derive impulse response functions for \( a, i, c, y, n \) for the model above.
(iii) Replace the technology process with

\[ a_t = \rho a_{t-1} + \epsilon_{t-3}. \]

Derive impulse response functions for \( a, i, c, y, n \) for the new model.

(iv) Try to change the elasticity of intertemporal substitution \( \sigma \) and see how it affects equilibrium dynamics.

(v) (OPTIONAL) Introduce quadratic adjustment costs in labor inputs:

\[ G(N_{t+1}, N_t) = \frac{\xi}{2} \left( \frac{N_{t+1} - N_t}{N_t} \right)^2. \]

Characterize the equilibrium dynamics for different values of \( \xi \).
Problem 2

Consider an economy where productivity follows the process

\[ x_t - x_{t-1} = \rho (x_{t-1} - x_{t-2}) + \epsilon_t. \]

Agents observe all past values \( \{x_{t-1}, x_{t-2}, \ldots\} \) and a signal regarding the current shock

\[ s_t = \epsilon_t + \epsilon_t. \]

Suppose consumers follow the forward-looking rule

\[ c_t = E \left[ \sum_{j=0}^{\infty} \beta^j x_{t+j} \mid \mathcal{J}_t \right], \]

where \( \mathcal{J}_t \) is the consumers' information set.

(i) Derive equilibrium consumption dynamics in terms of the shocks \( \epsilon_t \) and \( \epsilon_t \).

(ii) Suppose the econometrician information set at time \( t \), \( \mathcal{J}_t^E \), is given by \( \{x_{t-1}, x_{t-2}, \ldots\} \) and \( \{c_t, c_{t-1}, \ldots\} \). Write down a VAR representation for the joint behavior of \( x_{t-1} \) and \( c_t \):

\[
\begin{pmatrix} c_t \\ x_{t-1} \end{pmatrix} = \sum_{j=1}^{\infty} A_j \begin{pmatrix} c_{t-j} \\ x_{t-1-j} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \end{pmatrix}.
\]

(Hint: careful when defining the innovation to the \( x_{t-1} \) equation, notice that \( E [\epsilon_t | \mathcal{J}_t^E] \neq 0 \). Argue that the econometrician can identify \( s_t \) but cannot separately identify \( \epsilon_t \) and \( \epsilon_t \) from \( \{\eta_{1,t}, \eta_{2,t}\} \).

(iv) Suppose now the econometrician information set is \( \{x_t, x_{t-1}, \ldots\} \) and \( \{c_t, c_{t-1}, \ldots\} \). Write down a VAR representation for the joint behavior of \( x_t \) and \( c_t \):

\[
\begin{pmatrix} c_t \\ x_t \end{pmatrix} = \sum_{j=1}^{\infty} A_j \begin{pmatrix} c_{t-j} \\ x_{t-j} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \end{pmatrix}.
\]

Discuss how an econometrician can impose identifying restrictions to estimate and recover the shocks \( \epsilon_t \) and \( \epsilon_t \) from the innovations \( \{\eta_{1,t}, \eta_{2,t}\} \).

Problem 3

Consider an economy populated by a continuum of households \([0, 1]\) located on different islands.

Each household has an endowment \( \bar{x} = 1 \) of gold. Each household is made of a consumer and a producer. At the beginning of the day the producer sets the price \( p_i \). Then the consumer \( i \) travels to an island \( j \), randomly assigned. Then the preference shock \( \alpha_i \) is realized. The consumer observes \( \alpha_i \) and buys \( c_i \) units of the good produced in island \( j \). At the same time the producer is selling \( y_i \).
to some other consumer. Then the consumer returns home and consumes the gold:

\[ x_i = \bar{x} - p_j c_i + p_i y_i. \]

The central imperfection is that agents do not observe \( y_i \) (sales) at the time of making the purchases \( c_i \).

Preferences are as follows

\[ E \left[ u(c_i, \alpha_i) + w(x_i) - v(n_i) \right] \]

where \( c_i \) is consumption, \( x_i \)

\[
\begin{align*}
u(c_i, \alpha_i) &= \alpha_i c_i - \frac{1}{2} c_i^2 \\
w(x_i) &= x_i - \frac{1}{2} x_i^2
\end{align*}
\]

and \( v(n_i) \) is a convex function.

The production function in each island is linear and given by:

\[ y_i = n_i. \]

For simplicity, let \( c_i, x_i \) and \( n_i \) vary in \((\infty, +\infty)\), and disregard all non-negativity constraints.

The preference shocks are generated by:

\[ \alpha_i = \alpha + \epsilon_i \]

where \( \alpha \) and \( \epsilon_i \) are independent gaussian random variables with mean zero and variances \( \sigma_\alpha^2 \) and \( \sigma_\epsilon^2 \) and \( \int \epsilon_i d\epsilon_i = 1. \)

Consider a symmetric equilibrium where \( p_i = p. \) For the purpose of this exercise we will fix \( p \) (i.e. disregard the optimality condition for prices at the beginning of the period).

(i) Write down the consumer first order condition and derive the optimal choice of \( c_i \) as a function of \( \alpha_i \) and \( E_i [y_i] \).

(ii) Show that \( p \) determines the degree of strategic complementarity in spending. Comment.

(iii) Find the equilibrium output \( y \) in the case of perfect information.

(iv) Go back to the case where agents only observe \( \alpha_i \). Find a linear equilibrium of the type

\[ c = \psi \alpha \]

and show that \( \psi \) is larger for larger values of \( \frac{\sigma_\psi^2}{\sigma_\alpha^2} \).

Consider the case where agents can observe both \( \alpha_i \) and a public signal of the preference shock

\[ s = \alpha + \epsilon \]
where $c$ is gaussian, independent of $\alpha$ and $\epsilon_i$, with mean zero and variance $\sigma_c^2$.

(v) Characterize an equilibrium of the type

$$c = \psi_\alpha \alpha + \psi_\epsilon \epsilon$$

(Please use the notation: $E[\alpha|\alpha_i, s] = \beta_\epsilon \alpha_i + \beta_\epsilon s$)

(vi) Show that for a given value of $p$—the economy is very responsive to the public signal shock $e$ when $\beta_\epsilon$ is large and $\beta_\alpha$ is small.

(vii) Comment on the welfare implications, is the presence of the signal $s$ always desirable?

**Problem 4**

Consider the version of the Lucas (1972) model derived in class.

(i) Derive an expression for the constant $\xi$ or (which is the same) for the average price level $\bar{p}$. (Hint: you can take unconditional expectations on both sides of the labor supply equation to get

$$E[N_{i,t}] = E \left[ \frac{P_{i,t}}{P_{j,t+1}} (1 + x_{t+1}) \right],$$

substitute the equilibrium prices...)

(ii) Study the effect of changing $\sigma_c^2$ on average labor supply and average output, interpret.

(iii) (OPTIONAL) Consider a planner who uses a utilitarian welfare function (i.e. who maximizes $E \left[ \int C_{i,t} dt - \frac{1}{2} \int N_{j,t}^2 dt \right]$ each period). What is the level of $\sigma_c^2$ that maximizes welfare?