1.3 Nominal rigidities

- two period economy
- households of consumers-producers
- monopolistic competition, price-setting
- uncertainty about productivity
• preferences

\[ \sum_{t=1}^{2} \beta^t \left( \log C_{it} - \frac{\kappa}{1 + \eta} N_{it}^{1+\eta} \right), \]

\( C_{it} \) is the CES aggregate

\[ C_{it} = \left( \int_{0}^{1} C_{ijt}^{\sigma} \, di \right)^{\frac{\sigma}{\sigma-1}}, \]

with \( \sigma > 1 \)

• Technology

\[ Y_{it} = A_t N_{it}. \]
• productivity shocks $A_t$

$$A_t = e^{a_t}$$

$$a_1 = x + \epsilon_1,$$
$$a_2 = x + \epsilon_2$$

• $x$ and $\epsilon_t$ mean-zero, i.i.d., normal

• A signal about long-run productivity

$$s = x + e$$
• nominal balances with central bank at nominal rate $R$

• household set $P_{it}$ then consumers buy

• intertemporal BC

\[ (P_2 C_{i2} - P_{i2} Y_{i2}) + R \cdot (P_1 C_{i1} - P_{i1} Y_{i1}) \leq 0, \]

• $P_t$ is the price index

\[ P_t = \left( \int P_{it}^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}}. \]
Flexible price equilibrium

- period 2. Optimality for price-setting,

\[(1 - \sigma) \frac{1}{P_tC_{it}} \frac{P_{it}Y_{it}}{P_{it}} + \kappa\sigma \frac{1}{A_t P_{it}} Y_{it} N_{it}^{\eta} = 0.\]

- symmetric equilibrium, \(Y_{t} = A_t N_t\), this condition gives

\[N_t = \left(\frac{\sigma - 1}{\kappa\sigma}\right)^{\frac{1}{1+\eta}} = 1\]

(normalization of \(\kappa\)).

- quantities

\[C_t = Y_t = A_t.\]
• what about consumers’ decisions?

• consumer Euler equation

\[
\frac{1}{C_1} = RE \left[ \frac{P_1}{P_2 C_2} \right| a_1, s]
\]

• \(C_t = A_t\) log-normal

\[
r + p_1 - E[p_2|a_1, s] = E[a_2|a_1, s] - a_1 - \frac{1}{2} Var[a_2|a_1, s].
\]

• all changes in \(E[y_2]\) go to the real interest rate

• notice role of \(p_1\): neutralizes \(r\)
Fixed prices in period 1

- price-setting before any shock observed

\[ E \left[ (1 - \sigma) \frac{1}{P_1 C_{i1}} \frac{P_{i1} Y_{i1}}{P_{i1}} + \kappa \sigma \frac{1}{A_2} N_{i1}^\eta \frac{Y_{i1}}{P_{i1}} \right] = 0. \]

- rearranging this gives

\[ E \left[ N_1^{1+\eta} \right] = 1 \]

- this will pin down averages but not responses to shocks
• quantities: equilibrium in period 2 identical

• in period 1 now Euler equation (set $p_2 = 0$)

$$c_1 = E [a_2|a_1, s] - \frac{1}{2} Var [a_2|a_1, s] - r - p_1.$$ 

• suppose $r$ fixed, $p_1$ fixed by assumption

• now “sentiment shocks” affect consumption
Figure 8: RBC and Simple Monetary Model
Expectation of Technology Shock in Period 13 Not Realized

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• pin down $p_1$

$$E \left[ N_1^{1+\eta} \right] = E \left[ e^{(1+\eta)(y_1-a_1)} \right] = 1,$$

• thanks to log-normality this equation can be solved explicitly and gives

$$-r - p_1 - \frac{1}{2} \text{Var} [a_2|a_1, s] + \frac{1}{2} (1 + \eta) (\beta + \delta - 1)^2 \sigma_x^2 +$$

$$+ \frac{1}{2} (1 + \eta) (\beta - 1)^2 \sigma_\epsilon^2 + \frac{1}{2} (1 + \eta) \delta^2 \sigma_\epsilon^2 = 0$$

• where

$$E [a_2|a_1, s] = \beta a_1 + \delta s$$
• simple implication anticipated changes in \( r \) are neutral

• if instead we follow rule, e.g.

\[
r = \alpha_0 + \alpha_1 y_1
\]

then economy response changes

• we’ll go back to monetary policy
What about demand shocks?

- here price response is absent

- need a bit more flexibility
  - sticky prices
  - imperfect information
1.4 Lucas-Phelps islands

- Lucas 1972

- Overlapping generations

- Agents work at date $t$ consume at date $t + 1$

- Preferences

\[ E \left[ C_{i,t+1} - \frac{1}{2} N_{i,t}^2 \right] \]
• money
$x_t$ proportional subsidy from gov't

agents work, accumulate money, spend, die

\[
\begin{align*}
Y_{i,t} &= N_{i,t} \\
M_{i,t+1} &= P_{i,t}Y_{i,t}(1 + x_{t+1}) \\
P_{j,t+1}C_{i,t+1} &= M_{i,t+1}
\end{align*}
\]

at date $t + 1$ agent $i$ consumes the output of agent $j$
• continuum of islands, \( i \in [0, 1] \)

• unit mass of agents on each

• old agents receive proportional transfer \( x_t \) from govt'

• they travel to one island where they spend all their money

• prices \( P_{i,t} \) determined in walrasian equilibrium

• young agents decide their labor supply only observe \( P_{i,t} \)
• old agents in island $i$ are representative sample

• but different mass $\phi_{i,t}$

• nominal demand demand in island $i$ is

$$\phi_{i,t} \int_0^1 M_{i,t} di = \phi_{i,t} M_t$$

• $\phi_{i,t}$ log-normal with

$$\int_0^1 \phi_{i,t} di = 1$$
• Idiosyncratic demand shock

\[ \log \phi_{i,t} = u_{i,t} \]

• Monetary shocks log-normal

\[ \epsilon_t = \log (1 + x_t) \]

• Total nominal demand is

\[ D_{i,t} = \phi_{i,t} (1 + x_t) M_{t-1} \]

in logs

\[ d_{i,t} = \epsilon_t + u_{i,t} + m_{t-1} \]
Market clearing

\[ P_{i,t}N_{i,t} = \phi_{i,t} (1 + x_t) M_{t-1} \]
Information structure

- all agents observe \( \{M_{t-1}, M_{t-2}, \ldots \} \)

- old agents observe \( x_t, P_{j,t} \) (\( j \) is the good they buy)

- young agents observe \( P_{i,t} \)

observing \( P_{i,t} \) and \( M_{t-1} \), and knowing their own \( N_{i,t} \) young agents can infer

\[
\phi_{i,t} (1 + x_t)
\]
Labor supply

Agents solve

\[
\begin{align*}
\max_{N_{i,t}, C_{i,t+1}} \quad & E \left[ C_{i,t+1} - \frac{1}{2} N_{i,t}^2 | P_{i,t}, M_{t-1} \right] \\
\text{s.t.} \quad & P_{j,t+1}C_{i,t+1} = P_{i,t}N_{i,t} \left( 1 + x_{t+1} \right)
\end{align*}
\]

Substitute \( C_{i,t+1} \) and obtain

FOC

\[
E \left[ \frac{P_{i,t}}{P_{j,t+1}} (1 + x_{t+1}) - N_{i,t} | P_{i,t}, M_{t-1} \right] = 0
\]
interpretation

\[ N_{i,t} = \text{E} \left[ \frac{P_{i,t}}{P_{j,t+1}} (1 + x_{t+1}) | P_{i,t}, M_{t-1} \right] \]

labor supply exp.infl.
Equilibrium prices

guess:

\[ P_{i,t} = g(\phi_{i,t} (1 + x_t)) M_{t-1} \]

Because \( \phi_{i,t} \) and \( x_t \) are i.i.d. the distribution of \( \phi_{j,t+1} (1 + x_{t+1}) \) is given at date \( t \).

Decompose

\[
N_{i,t} = E\left[\frac{P_{i,t}}{P_{j,t+1}} (1 + x_{t+1}) | P_{i,t}, M_{t-1}\right] =
\]

\[
= E_{i,t} \left[\frac{P_{i,t}}{M_t}\right] E_{i,t} \left[\frac{M_t}{P_{j,t+1}} (1 + x_{t+1})\right]
\]
then

$$E \left[ \frac{1 + x_{t+1}}{g \left( \phi_{j,t+1} (1 + x_{t+1}) \right)} \phi_{j,t+1} (1 + x_{t+1}) \right] = \xi$$

is a constant independent of today’s shocks

$$N_{i,t} = \xi E_{i,t} \left[ \frac{P_{i,t}}{M_{t-1}} \frac{1}{1 + x_t} \right]$$
From equilibrium condition we obtain

$$\phi_{i,t} \frac{M_t}{P_{i,t}} = N_{i,t} = \xi E_{i,t} \left[ \frac{P_{i,t}}{M_t} \right]$$

in logs,

$$m_t - p_{i,t} + u_{i,t} = (\ldots) - E_{i,t} \left[ m_t - p_{i,t} \right]$$

(constant terms in (\ldots), depend on variances)

We obtain

$$p_{i,t} = \bar{p} + \frac{1}{2} \left( m_t + u_{i,t} \right) + \frac{1}{2} E_{i,t} \left[ m_t \right]$$
Agents observe

\[ m_t + u_{i,t} = m_{t-1} + \epsilon_t + u_{i,t} \]

Define

\[ E_t [m_t] = \int_0^1 E \left[ m_t | m_{t-1}, \epsilon_t + u_{i,t} \right] di \]

Then, averaging, we have

\[ p_t = \bar{p} + \frac{1}{2} m_t + \frac{1}{2} E_t [m_t] \]
Imperfect information

\[ E_t [m_t] \neq m_t \]

in particular

\[ E \left[ m_t | m_{t-1}, \epsilon_t + u_{i,t} \right] = m_{t-1} + \beta (\epsilon_t + u_{i,t}) \]

where

\[ \beta = \frac{\sigma_{\epsilon}^2}{\sigma_{m}^2 + \sigma_{u}^2} \]

so

\[ E_t [m_t] = m_{t-1} + \beta \epsilon_t \neq m_{t-1} + \epsilon_t \]
We have
\[ p_t = \bar{p} + m_{t-1} + \frac{1}{2} (1 + \beta) \epsilon_t \]

and output is
\[ y_t = m_t - p_t = \bar{y} + \frac{1}{2} (1 - \beta) \epsilon_t \]

- larger $\frac{\sigma_m^2}{\sigma_u^2}$ implies smaller real effects of monetary policy
- Phillips curve depends on the monetary regime
Wrapping up

- with partially revealing prices
  - first order expect. \( m_t \neq \overline{E}_t [m_t] \)

- this can explain short-run non-neutrality

- prices adjust less than 1:1 with imp. info

- policy regime affects inference and thus effects of shocks