14.471: Fall 2012: Recitation 12: Elasticity of Intertemporal Substitution (EIS)

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1 Hall (1987)

1.1 Goal, test and implementation challenges

- Goal: estimate the EIS $\sigma$ (the elasticity of the consumption ratio to the corresponding price ratio) with macro-data

- Tests the following relationship between the expected value of log of consumption in $t$ $\bar{c}_t$ given consumption in period $t-1$ $c_{t-1}$ and the mean of the distribution of the real interest rate (obtained in $t$ when investing in $t-1$) $\bar{r}_{t-1}$:

$$\bar{c}_t = \sigma \bar{r}_{t-1} + c_{t-1} + k$$

1. Assumptions to get equation (1):
   - intertemporally separable utility function
   - lognormality of consumption and
   - normality of the interest rate

- The actual log of consumption in $t$ $c_t$ is also influenced by unpredictable surprises to interest rates $v_t$ and consumption $\epsilon_t$:

$$c_t = \sigma \bar{r}_{t-1} + c_{t-1} + k + \epsilon_t = \sigma (\bar{r}_{t-1} - v_t) + c_{t-1} + k + \epsilon_t$$

2. Prediction:
   - Info available at $t-1$ is helpful in predicting the growth rate of consumption only to the extent that it predicts the real interest rate

3. 4 questions for implementation test:
   1. How to measure the expectations of the real interest rate $\bar{r}_{t-1}$?
   2. How to deal with the fact that we only have data on time-averages of consumption versus continuous/more frequent asset return data?
   3. For which assets to measure the real interest rate?
   4. Which time period to look at?
1.2 How to measure expectations of the real interest rate?

- Two approaches:
  - Use survey data on expected price changes: subtract taxes and expected price change from nominal rate
  - Relate the mean of the distribution of the real interest rate (obtained in \( t \) when investing in \( t - 1 \)) \( \bar{r}_{t-1} \) to variables \( x_{t-1} \) known to consumer when they pick \( c_{t-1} \):
    \[
    \bar{r}_{t-1} = x_{t-1} \beta
    \] (3)

- Think of (3) as instrumental variable estimation of consumption equation where the determinants of the expected real interest rate are the instruments

1.3 Time aggregation

- (2) applies to consumption in discrete/continuous time but does not characterize behavior of time averages of consumption
- Interest rate and rates of inflation are measured monthly or more frequently while only yearly consumption averages are available
- The issues with the relation among the time aggregates (denote the change in aggregate consumption by \( \Delta c_t \)) \( \Delta c_t = \sigma \bar{r}_{t-1} + \epsilon_t \) are:
  - \( \epsilon_t \) is not white noise but obeys a First-Order MA process with serial correlation of 0.25
  - \( \epsilon_t \) is correlated with \( r_{t-1} \) or with its instruments even if they are uncorrelated at the monthly level
- Use Hayashi and Sims (1983) estimator to deal with correlation in residuals and endogenous instruments (transformation of model to one without serial correlation, while keeping the instrument predetermined)
- Critical timing of instruments:
  - If data measured instantaneous consumption at 2 isolated points, any variable known at time when consumer picks \( c_{t-1} \) would be eligible as an instrument
  - But if \( c_{t-1} \) is an annual average, then any variable measured during calendar year \( t - 1 \) can be correlated with disturbance \( \epsilon_t \)
  - Thus: annual aggregates for year \( t - 2 \) are usable but not those for year \( t - 1 \)

1.4 For which assets?

- Treasury bills, bonds and stock market indices are all plausible but in reality a lot of heterogeneity in asset holdings across households
- Earlier evidence of high EIS based on fixed-income securities but much more variation in stock market returns.
1.5 Results

- Figure (1.5) suggests small EIS: change in consumption around 3% despite large variation in real interest rates

Figure 1: Real interest rate and changes in consumption

- Survey on inflation and stock price expectations:
  - Stock market returns: precise zero
  - Saving accounts, T-bills: lack of precision

- Annual changes in consumption since 1924, T-bills and instrumental variable estimation
  - Slightly negative EIS point estimate and relatively precise

- Reconciliation with Summers (1982) and Hansen and Singleton (1983):
Again very small estimates (<0.10) in contrast with previous work

Differences arise from:

* Longer time period (postwar)
* Elimination of inappropriate instruments
* Elimination of serial correlation induced by the time aggregation

2 Gruber (2006)

• Observation:

  – So far, the impact of the interest rate on consumption or savings is identified by time series movements in interest rates
  – But the factors that cause time series movements in interest rates may themselves be correlated with consumption or savings decisions

• Strategy:

  1. Use variation across individuals in the capital income tax rate and CES data over 2 decades
  2. To surmount the problem of determination of the tax rate as a function of income and savings decision, create “simulated” tax rates:
     (a) tax rates are only based on exogenous characteristics such as education, age and sex
     (b) control for these exogenous characteristics in the regression
     (c) Hope: effect of taxes is identified only by changes in the tax system over the period 1980 to 2001

• Finding: EIS of 2

2.1 Data

• Consumption from CES:

  – 5-10,000 households (HH’s) /year
  – quarterly survey since 1980
  – Survey each HH for up to 5 quarters

• Non-durable real consumption: total consumption minus durables and transfer spending

• Pre-tax returns from CSF:

  – Compute income-group specific distribution of asset holdings
  – take WA of after-tax rates of return for each type of assets (assuming marginal=average)

• Transform pre-tax return into after-tax rate of return using TAXSIM model for each household

  – compute MTR on interest income, dividend income and capital gains income
  – State level variation in interest and dividend income
  – tax rates on capital gains differ because tax preferences

• Combine relevant tax rate at household level with relevant asset return
2.2 Empirical framework

- Estimate a standard log-linearized Euler equation:

\[ CG_{it,t+1} = \alpha + \beta ATRATE_{it} + X_{it} \delta + \Delta Z_{it,t+1} \eta + \epsilon \]  

- \( ATRATE_{it} \): the after-tax interest rate that applies to savings between \( t \) and \( t+1 \)
- \( \Delta Z_{it,t+1} \): changes in demographics
- 3 potential issues with (4):
  1. Much of the variation in the after-tax interest rate is time series variation
  2. Cross-sectional variation in both taxes and pre-tax returns is driven largely by income differences. Income is a function of tastes for consumption through capital-income and hence bias.
  3. Non-capital income differences are likely correlated with omitted determinants of both the level and growth rate of consumption, such as consumption growth uncertainty

Proposed solution to issue 1

- Instrument the after-tax rate of return with the tax rate on interest income, while controlling for a full set of year dummies
- \( \rightarrow \) use variation in tax rates across individuals not time series movements in interest rates

Proposed solution to issue 2

- Use predicted income rather than actual income for HH
- Predictors: marital status, education, age, race, state, ...
- \( \rightarrow \) Tax rate measure with is independent of unobserved tastes for consumption

Proposed solution to issue 3

- Predicted tax rate is still a function of observed factors which may also be correlated with the taste for consumption
- Thus include a set of:
  - linear controls for each of these factors
  - 100 dummies for each point in income distribution \( \rightarrow \) identification does not come from cross-sectional differences in income
- Fixed state effects
- Thus: model is identified by changes in state taxes that deviate from the national trend, or changes in national taxes that have differential impacts along the income distribution.
2.3 Results

- Table (2.3) suggests:
  - a high sensitivity of OLS rate of return, or IV lagged rate of return estimates to the definition of the security used to compute the return
  - high point estimates in the range of 2 with the preferred tax IV specification
  - relatively low precision of the preferred tax IV specification

Table 1: Table 2 from Gruber (2006)

<table>
<thead>
<tr>
<th></th>
<th>After-tax T-Bill Rate</th>
<th>After-tax Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS, no year dummies</td>
<td>-0.551 (0.116)</td>
<td>0.105 (0.032)</td>
</tr>
<tr>
<td>Lag IV, no year dummies</td>
<td>2.616 (0.490)</td>
<td>0.328 (0.130)</td>
</tr>
<tr>
<td>Tax IV</td>
<td>2.032 (0.796)</td>
<td>2.239 (0.894)</td>
</tr>
<tr>
<td>Number Obs</td>
<td>66314</td>
<td>66208</td>
</tr>
</tbody>
</table>

Notes: Estimates from models such as equation (1) in text. Each cell represents the estimated EIS from a separate model: first column uses after-tax T-bill rate, while second column uses weighted average after-tax rate of return. Standard errors in parentheses.