14.471: Fall 2012: Recitation 7: Application of linear taxation to intertemporal taxation

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October 26, 2012

Questions: How to set optimal taxes on labor and capital in a dynamic infinite horizon economy? How do the results change when we allow for heterogeneity across agents and poll taxes?

1 Review of lecture notes

1.1 Results and main intuition:

We have 3 results:

1. At the steady state, the tax on capital is zero.
2. Initial tax on capital and bonds (lump sum expropriation but time inconsistency)
3. Labor tax smoothing

Intuition for the zero capital tax result (see Salanié p. 140) Assume that at the steady state capital is paid a before-tax return \( r \) and its tax rate is \( \tau \). Then capital taxation changes the relative price of consumption at date \( t \) and date \( t+T \) by a factor:

\[
\left( \frac{1+r}{1+r(1-\tau)} \right)^T
\]

Indeed, without taxes consuming 1 today (at \( t \)) costs \((1+r)^T\) in terms of forgone consumption at \( T+t \). But with a tax consuming 1 today costs\((1+r(1-\tau))^T\) of forgone consumption at \( T+t \). Hence if the tax rate \( \tau \) is positive and when \( T \) tends to infinity, then the relative price of consuming today becomes zero! We get massive intertemporal distortion and incentives to consume today. Note: in reality, real-world consumers do not live infinite lives.

1.2 Setup

- Preferences: \( \sum \beta^t u(c_t, L_t) \)
- Resource constraint:
  \[c_t + g_t + k_{t+1} \leq F(k_t, L_t) + (1-\delta)k_t \]
- Define the agent’s period-by-period budget constraint affected by linear taxes:
  \[c_t + k_{t+1} + q_{t,t+1}B_{t+1} \leq (1-\tau_t)w_tL_t + R_tK_t + (1-k_t^B)B_t \]
where \( q_{t,t+1} \) is the price of a bond at \( t \) paying out \$1 at \( t+1 \), \( R_t = 1 + (1 - \kappa_t)(r_t - \delta) \) is the gross after tax return net of depreciation, consumption is not taxed (normalization) and WLOG we have a zero tax on bonds after the 1st period \( \kappa_1^B = 0 \) \( t > 0 \) (if we were to tax bonds, then the bond prices would simply drop).

- The no-ponzi conditions:
  - \( q_{0,t} = q_{0,1}q_{1,2}...q_{t-1,t} \): the cost of buying 1 unit of consumption at \( t \) should be the same whether you buy a bond with maturity \( t \) today or buy 1 period-bonds which you then reinvest every period until \( t \)
  - \( \lim q_{0,t}B_T \geq 0 \): The discounted value of bond holdings at infinity cannot be negative.
- Budget constraint government: \( g_t + B_t \leq \tau_tw_tL_t + \kappa_tK_br_t + q_{t,t+1}B_{t+1} \)
- Define an equilibrium where (i) agents maximize given prices and taxes, (ii) firms chose labor and capital inputs to maximize profits, (iii) government satisfies its B.C. and (iv) good-, capital- and bond markets clear

### 1.3 Methodology/Primal Approach:

- Write the following NPV budget constraint for the agent as a function of bond prices, initial holdings, consumption, labor but without capital (tricks: Solve bond holdings \( B_t \) forward and eliminate capital with “no-arbitrage”):
  \[
  \sum_{t=0}^{\infty} q_{0,t} (c_t - (1 - \tau_t)w_tL_t) \leq R_0K_0 + (1 - \kappa_0^B)B_0 \tag{1}
  \]

- 2 Tricks to get (1):
  - Solving \( B_t \) forward:
    1. Use the budget constraint of the agent at \( t = 0 \):
       \[
       c_0 + k_1 + q_{0,1}B_1 - (1 - \tau_0)w_0L_0 - R_0K_0 \leq (1 - \kappa_0^B)B_0 \tag{2}
       \]
    2. Use the budget constraint of the agent at \( t = 1 \) to solve \( B \) forward and use that \( \kappa_1^B = 0 \)
       \[
       c_1 + k_2 + q_{1,2}B_2 - (1 - \tau_1)w_1L_1 - R_1K_1 = B_1 \tag{3}
       \]
  3. Combining (2) and (3) gives:
     \[
     c_0 + k_1 + q_{0,1} \{c_1 + k_2 + q_{1,2}B_2 - (1 - \tau_1)w_1L_1 - R_1K_1\} - (1 - \tau_0)w_0L_0 - R_0K_0 \leq (1 - \kappa_0^B)B_0
     \]
  4. Repeating this we get that the NPV of “net consumption and investment above earnings” cannot exceed initial bond holdings:
     \[
     \sum_{t=0}^{\infty} q_{0,t} \{c_t + k_{t+1} - (1 - \tau_t)w_tL_t - R_tK_t\} \leq (1 - \kappa_0^B)B_0 \tag{4}
     \]
    - Eliminating capital from (4) using no arbitrage:
      1. Regrouping the terms in (4) with \( k_2 \) gives \( q_{0,1}k_2 - q_{0,2}R_2k_2 \)
      2. Now use that \( q_{0,t} = \frac{2R_{t-1}}{K_0} \) (the cost of obtaining 1 dollar at \( t \) should not depend on whether you buy a long-term bond or buy a medium-term bond and reinvest it later in a 1 period bond)
- Combine the agent’s NPV budget constraint (1) and his consumption and leisure FOC’s into the implementability condition (trick: pricing equation \( q_{0,t} = \beta^t \)):
  \[
  \sum_{t=0}^{\infty} \beta^t (u_{c,t}c_t + u_{L,t}L_t) \leq u_{c,t} \left( R_0K_0 + (1 - \kappa_0^B)B_0 \right) \tag{5}
  \]
1. FOC’s wrt \( c_t \) and \( L_t \) when consumer maximizes \( \sum \beta^t u(c_t, L_t) \) s.t. (1) give us:
   (a) \( \beta^t u_c = \lambda q_{0,t} \)
   (b) \( \beta^t u_l = -\lambda q_{0,t}(1 - \tau_t) w_t \)
   (c) Combining we get the intratemporal condition for the agent:
   \[
   w_t (1 - \tau_t) = - \frac{\mu_t}{u_c}
   \]
   (6)
   (d) Using that \( \frac{\beta^t u_c(c_t, L_t)}{q_{0,t}} = \lambda = \frac{\beta^{t+1} u_c(c_{t+1}, L_{t+1})}{q_{0,t+1}} \) and the no-arbitrage condition, we get the intertemporal condition:
   \[
   \beta R_{t+1} u_c(c_{t+1}, L_{t+1}) = u_c(c_t, L_t)
   \]
   (7)

2. Now you multiply (1) with \( u_c \) to get (5)
   - Let the planner maximize expected utility of the consumer s.t. implementability (5) and the resource constraint \( F(k_t, L_t) + (1 - \delta)k_t = c_t + g_t + k_{t+1} \) where \( W(c, L, \mu) = u(c, L) + \mu (u(c, l)c + u_L(c, L)L) \) and get intra and intertemporal “ish” conditions for \( W \) where the social rate of return equals \( R_{t+1}^* = F_k(k_{t+1}, l_{t+1}) + 1 - \delta \)
     - Creating the Lagrangian:
       * Indeed, the Lagrangian of the consumer (his objective function and his implementability condition) is:
         \[
         L = \sum \beta^t u(c_t, L_t) + \mu \left\{ \sum_{t=0}^{\infty} \beta^t (u_{c,t} + u_{L,t} L_t) - u_{c,t} (R_0 K_0 + (1 - \kappa_0 B_0)) \right\}
         \]
       * Rewrite this as
         \[
         L = \sum \beta^t [u(c_t, L_t) + \mu (u_{c,t} + u_{L,t} L_t)] - \mu u_{c,t} (R_0 K_0 + (1 - \kappa_0 B_0))
         \]
       \[
       L = \sum \beta^t W(c, L; \mu) - \mu u_{c,t} (R_0 K_0 + (1 - \kappa_0 B_0))
         \]
       * Hence the planner solves this maximization problem subject to the resource constraint:
         \[
         L_{\text{planner}} = \sum \beta^t W(c, L; \mu) - \mu u_{c} (R_0 K_0 + (1 - \kappa_0 B_0)) + \lambda_{RC} (F_k(k_t, L_t) + (1 - \delta)k_t - c_t - g_t - k_{t+1})
         \]
     - FOC’s to the planner’s maximization problem give:
       * Labor:
         \[
         \beta^t W_L = -\lambda_{RC} F_L
         \]
       * Capital:
         \[
         \lambda_{RC,t+1} (F_K + (1 - \delta)) = \lambda_{RC,t}
         \]
       * Consumption:
         \[
         \beta^t W_C,t = \lambda_{RC,t}
         \]
     - Combining the planner’s FOC conditions:
       * Combining the labor and consumption FOC’s gives:
         \[
         \frac{W_L(c_t, l_t; \mu)}{W_C(c_t, l_t; \mu)} = F_L(K_t, L_t) = -w_t
         \]
* Combining the capital and consumption FOC’s gives:

\[ \beta^t W_{C,t} = \beta^{t+1} W_{C,t+1} (F_K + (1 - \delta)) \]

\[ W_{C,t} = \beta W_{C,t+1} (F_K + (1 - \delta)) = \beta W_{C,t+1} R_{t+1}^* \]

where \( F_k(k_{t+1}, L_{t+1}) + 1 - \delta = R_{t+1}^* \) is the social rate of return

- Remember the intra- and intertemporal conditions (6) and (7) for the agent:

\[ w_t(1 - \tau_t) = -\frac{u_t}{u_c} \]

\[ \beta R_{t+1} u_c(c_{t+1}, L_{t+1}) = u_c(c_t, L_t) \]

- Hence, we can combine the planner’s and the agent’s optimality conditions to get insights on optimal taxes:

\[ 1 - \tau_t = -\frac{u_t}{u_c} \frac{1}{w_t} = \frac{W_c}{W_L} \frac{u_t}{u_c} \]

\[ \frac{R_{t+1}}{R_{t+1}^*} = \frac{u_c(c_t, L_t)}{u_c(c_{t+1}, L_{t+1})} \frac{W_c(c_{t+1}, L_{t+1}; \mu)}{W_c(c_t, L_t; \mu)} \]  

1.4 Results

- At the steady state, the tax on capital is zero (i.e. \( \kappa_t = 0 \) ) since:

\[ \frac{R_{t+1}}{R_{t+1}^*} = 1 = 1 + (1 - \kappa_t)(r_t - \delta) \]

- Initial tax on capital and bonds (lump sum expropriation but time inconsistency)
- Labor tax smoothing: no special role for current \( g_t \) conditional on current allocation but expenditures affect \( \mu \)

2 Highlights of Werning (2007): Extension to heterogeneous agents

2.1 Introduction

- Standard Ramsey model adopts a representative-agent framework and derives optimal taxes on labor and capital (Chamley-Judd) where the reason for distortionary taxation is the ruling out of lump-sum taxes.
- But poll taxes are realistic (e.g. tax deductions or transfers from welfare programs) and a more natural rationale for distortionary taxation is distributional concerns (Mirrlees 1971): for instance non observable differences in productivity.
- Here focus on linear taxation with a poll tax: summarize the labor-income tax schedule with the lump-sum tax \( T_t \) and the slope or marginal tax rate \( \tau_t \).
2.2 Differences and similarities in set-up (2)

Differences

- Finite types $i$ with weight $\pi_i$ and with different preferences $U^i(c_t, L_t)$:
  - typically differences in productivity $U^i(c, L) = U(c, \frac{L}{\pi_i})$:
- Type of workers is private information
- Uncertainty captured by a publicly observed state $s_t$ where the probability of a history $s^t$ is denoted $Pr(s^t)$
- $p(s^t)$ is the Arrow-Debreu price of consumption in period $t$ after history $s^t$
- Allow for a lump-sum tax (poll tax)

Similarity

- Definition of a competitive equilibrium

2.3 Differences in solution methodology

Fictitious agent

- Also primal approach: formulate planning problem in terms of aggregate allocation that can be implemented with taxes and prices (remember: in dual tax rates and prices are not eliminated but are the planner’s controls)
- With linear taxes, all workers face the same after-tax prices for consumption and labor:
  - marginal rates of substitution are equated across workers
  - all inefficiencies due to distortive taxation are confined to the determination of aggregate consumption and aggregate labor
- NEW: Equilibrium after-tax prices can be computed as if the economy were populated by a fictitious representative agent with the utility function

$$U^m(c, L, \varphi) = \max_{\{c_i, L_i\}} \sum \varphi^i U^i(c^i, L^i) \pi_i$$

where the weighted sums of individuals’ consumptions and labor levels equal the aggregate levels.

- Then, compute fictitious agent’s intertemporal and intratemporal optimality conditions, combine with budget constraint and get implementability condition

Planning problem

- Set of competitive equilibrium defines a set of attainable lifetime utilities
- Planner:
  - chooses aggregate levels of consumption, labor, capital, market weights and the lump-sum tax
  - wants to reach the northeastern frontier of the set of attainable lifetime utilities
- More equal weights imply a more equal consumption allocation and hence higher tax rates
- Also “pseudo-Lagrangian”
2.4 Some differences in terms of results

- The Chamley-Judd zero capital tax result is quite robust
- NEW 1: Distortionary taxation is a redistribution mechanism (since poll taxes are allowed)
  - A positive tax rate makes high-skilled, rich workers pay more taxes than low-skilled poor workers
  - The optimal tax rate balances distributional concerns against efficiency
- NEW 2: In some cases initial wealth taxation is unnecessary. If all workers start with the same capital holdings, then the effect of the initial capital levy is equivalent to a lump-sum tax
- NEW 3: Since capital levies can be become unnecessary, the time inconsistency problem result is not robust.
- NEW 4: If skill distribution changes over time, then tax smoothing results could fail since the trade-off between efficiency and distributional concerns becomes time-varying
14.471 Public Economics I
Fall 2012

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