Today’s Plan

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2 Homothetic Preferences
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   2 Properties
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The small graphs on slides 3-5, 7-19, 21, and 24-28 are courtesy of Marc Melitz. Used with permission.
Budget Set

- 2 goods: Cloth ($C$) and Food ($F$); Consumption level $D = (D_C, D_F)$
- Given prices $p_C$ and $p_F$ and income $I$
- Budget set is set of consumption bundles such that $p_C D_C + p_F D_F \leq I$

$p_C / p_F$ is the relative price of $C$ (measured in units of $F$)
In a trading environment, income is determined by value of endowment \( E = (E_C, E_F) \) (bundle of goods that can be traded)

So budget line is given by

\[
p_C D_C + p_F D_F = p_C E_C + p_F E_F \iff \frac{p_C}{p_F} D_C + D_F = \frac{p_C}{p_F} E_C + E_F
\]

\( \Rightarrow \) Only relative price \( p_C/p_F \) matters! (‘nominal’ prices are irrelevant)
Preferences

- Represented by a utility function $U(D_C, D_F)$
- Recall that utility is an ordinal concept, so units don’t matter (only ranking)
  - $U + a$, $aU$, $U^2$, $\sqrt{U}$, $\log U$, $e^U$ all represent the same preferences
- Marginal utility of each good are assumed to be non-negative:
  - $MU_C = \frac{\partial U(D_C, D_F)}{\partial D_C} \geq 0$ and $MU_F = \frac{\partial U(D_C, D_F)}{\partial D_F} \geq 0$
- Preferences are completely summarized by an indifference curve map $U(D_C, D_F) = \overline{U}$ for any $\overline{U}$:
Marginal Rate of Substitution

- At any point on an indifference curve, the marginal rate of substitution is defined as $MRS = \frac{MU_C}{MU_F}$
  - Important note: to avoid confusion, will always refer to MRS in absolute value (a positive number)
  - You may have seen it defined as $MRS = -\frac{MU_C}{MU_F}$
  - The $MRS$ at any consumption point is the slope of the tangent to the indifference curve at that point

- In words: $MRS$ is the amount of $F$ a consumer is willing to trade for one unit of $C$
  - That is, leaves the consumer on the same indifference curve (utility level remains constant)
  - It is the consumer’s valuation of a unit of $C$ –measured in units of $F$
  - The MRS captures the substitutability between $C$ and $F$ at the current consumption point
Further assumption on preferences: they are (weakly) convex
Indifference curves are bowed out to the origin
*MRS* is decreasing as consumption of *C* increases
The more *C* is consumed, the less valuable it becomes relative to *F*
Example: Linear Preferences

- \( U(D_C, D_F) = aD_C + bD_F \)

- Consumer is always indifferent between \( \Delta D_C = b \) and \( \Delta D_F = a \)
- \( MRS \) is constant at \( a/b \)
- What does this imply about the substitutability of \( C \) and \( F \)?
Example: Leontief Preferences

- $U(D_C, D_F) = \min \{aD_C, bD_F\}$
- Consumer always wants to consume $b$ units of $C$ with $a$ units of $F$
- $MRS$ is undefined
- What does this imply about the substitutability of $C$ and $F$?
Utility Maximization

At an interior optimum, $MRS = \frac{p_C}{p_F}$

Whenever $MRS > \frac{p_C}{p_F}$, consumer wants to trade $F$ for $C$

Whenever $MRS < \frac{p_C}{p_F}$, consumer wants to trade $C$ for $F$
Tangency of Budget Line and Indifference Curve at the Interior Optimum

Why is this a necessary condition?
$D_C = 0$ is an optimum if $MRS < \frac{p_C}{p_F}$ at that point. Why?

Consumer wants to trade $C$ for $F$, but there is no more $C$ left to trade!
$D_F = 0$ is an optimum if $MRS > \frac{p_C}{p_F}$ at that point. Why?

Consumer wants to trade $F$ for $C$, but there is no more $F$ left to trade!
Given preferences and endowment $E$, optimal (util. max) demand $D$ can be calculated for any given relative price $p_C/p_F$.
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![Diagram](image-url)
This pattern of demand can be represented as a relative demand curve i.e. $D_C / D_F$ as a function of $p_C / p_F$:

In general, a relative demand curve ($RD$) will depend on the consumer’s endowment point $E$. 

![Graph showing relative demand curve](image-url)
Homothetic Preferences

- **Definition:** MRS is constant along any ray from the origin.

- A single indifference curve summarizes all the information about preferences.
Changes in income are proportionally reflected in the optimal demand for all goods (holding prices fixed).

This leads to some very important aggregation properties across consumers with different income levels.
Special Examples of Homothetic Preferences

- Cobb-Douglas preferences: \[ U(D_C, D_F) = (D_C)^a(D_F)^b \] with \( a, b > 0 \)
  - Consumer always spends a constant share of his/her income on both goods:
    \[
    \frac{p_C D_C}{p_C D_C + p_F D_F} = \frac{a}{a + b} \quad \text{and} \quad \frac{p_F D_F}{p_C D_C + p_F D_F} = \frac{b}{a + b}
    \]

- Linear preferences
- Leontief preferences
If consumers have the same homothetic preferences, then they will always consume the same relative amount of $C$ and $F$ –regardless of differences in their endowments.

Thus, the $RD$ curve for any homothetic preferences is independent of the consumer’s endowment.
Consider $N$ consumers indexed by $i = 1..N$

For each consumer $i$:

$$pD_i^C + D_i^F = pE_i^C + E_i^F \quad \text{(budget constraint)}$$

where $p = p_C / p_F$ is the relative price

Now sum the budget constraints:

$$p \sum_{i=1}^{N} D_i^C + \sum_{i=1}^{N} D_i^F = p \sum_{i=1}^{N} E_i^C + \sum_{i=1}^{N} E_i^F \iff pD_C + D_F = pE_C + E_F$$

where $D = (D_C, D_F)$ is aggregate demand and $E = (E_C, E_F)$ is the aggregate endowment – over all $N$ consumers.

Also, $D_i^C / D_i^F = RD(p)$ for all consumers $i$ so this must also hold in the aggregate: $D_C / D_F = RD(p)$

$\Rightarrow$ Aggregate demand is the same as if it were generated by a single consumer who owns the aggregate endowment $E$ and shares the same homothetic preferences as the individual consumers.
Can capture all the properties of aggregate demand for a country by modeling the demand of a single consumer.

Furthermore, this aggregate demand is independent of the distribution of endowments (hence incomes) across consumers.

Important note: If the welfare of this aggregate consumer is increasing (or decreasing) then this will imply that overall welfare is also increasing (or decreasing).

But this does not mean that the welfare of all individual consumers is increasing (or decreasing).
Recall that any homothetic preferences can be exactly described by the associated relative demand curve (since it is independent of endowments)
Consider 2 consumers with different homothetic preferences (1 and 2):

Who likes $C$ relatively more?

Consumer 2 does: at same $p_C/p_F$, he/she will always demand relatively more $C$ ($D_C^1/D_F^1 < D_C^2/D_F^2$)
Consider 2 consumers with different homothetic preferences (1 and 2):

- Who likes C relatively more? What is main difference between preferences?
- Consumer 1 considers C and F to be relatively closer substitutes (than consumer 2 does) – his/her demand is more elastic
Some Additional Examples

- Consider 4 consumers with different homothetic preferences (1-4):

- What are the relative demands?
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