1. Consider an economy (Home) with 2 goods, Cloth ($C$) and Food ($F$). All consumers have same Cobb-Douglas utility function:

$$U(D_C, D_F) = (D_C)^a (D_F)^b$$

with $a, b > 0$. Aggregate endowments of $C$ and $F$ are given by $E_C = 100$ and $E_F = 200$.

(a) Show that:

$$\frac{p_C D_C}{p_C D_C + p_F D_F} = \frac{a}{a + b} \quad \text{and} \quad \frac{p_F D_F}{p_C D_C + p_F D_F} = \frac{b}{a + b}$$

(b) Let $p = p_C/p_F$. What is the relative demand $RD(p) = D_C/D_F$ at Home?

(c) What is the relative price $p^A$ at Home under autarky?

(d) Now consider a second economy (Foreign) with same Cobb-Douglas preferences but different aggregate endowments $E_C^* = 200$ and $E_F^* = 100$. What is the relative price $p^{A*}$ abroad under autarky?

(e) What is the relative price $p^T$ under free trade?

(f) What is the pattern of trade?

(g) Show that both countries gain from trade

2. Adam and Eve are stranded on a desert island. There are only two goods on the island: Apples ($A$) and Bananas ($B$). The utility functions of Adam and Eve are $U^{Adam}(D_A, D_B) = 3D_A + D_B$ and $U^{Eve}(D_A, D_B) = D_A + 3D_B$, respectively. Total endowments on the island are 20 Apples and 60 Bananas. Adam owns all the bananas and Eve all the apples.

(a) Draw the Edgeworth box for this exchange economy, including Adam and Eve’s indifference curves and endowments.

(b) Using a graphical analysis, determine the contract curve of this economy.

(e) Will Adam consume any apple in a competitive equilibrium?

3. There are 2 goods, Cloth ($C$) and Food ($F$). Show that if preferences are homothetic, than a consumer with $n$ times the income of another will consume $n$ times more $C$ and $F$. 

