14.581 International Trade
Class notes on 3/20/2013

1 Intensive and Extensive Margins in Trade Flows

- With access to micro data on trade flows at the firm-level, a key question to ask is whether trade flows expand over time (or look bigger in the cross-section) along the:
  - Intensive margin: the same firms (or product-firms) from country $i$ export more volume (and/or charge higher prices—we can also decompose the intensive margin into these two margins) to country $j$.
  - Extensive margin: new firms (or product-firms) from country $i$ are penetrating the market in country $j$.

- This is really just a decomposition—we can and should expect trade to expand along both margins.

- Recently some papers have been able to look at this.
  - A rough lesson from these exercises is that the extensive margin seems more important (in a purely ‘accounting’ sense, not necessarily a causal sense).

\[\begin{array}{c|c|c|c}
\text{Country and Aggregate U.S. Exports, 2000} & \\
\hline
\text{Log of total exports value} & \text{Log of number of exporting firms} & \text{Log of number of exported products} & \text{Log of export value per product per firm} \\
\hline
\text{Log of GDP} & 0.89 & 0.71 & 0.62 & 0.25 \\
\text{Log of distance} & -0.36 & -1.84 & -1.84 & -0.84 \\
\text{Gallwarner} & 0.05 & 0.06 & 0.06 & 0.10 \\
\hline
\end{array}\]

Source: Data are from the 2000 Limited Longitudinal Firm Trade Transaction Database (LLTID).
Notes: Each column reports the results of a country-level ordinary least squares regression of the dependent variable listed in the top of each column on the variables listed in the first column. Robust standard errors are reported below each coefficient. Products are defined as eight-digit Harmonized System categories. All results are statistically significant at the 1 percent level.


\[\text{The notes are based on lecture slides with inclusion of important insights emphasized during the class.}\]
Table 9
Gravity and Aggregate U.S. Imports, 2000

<table>
<thead>
<tr>
<th>Log of total import value</th>
<th>Log of number of exporting firms</th>
<th>Log of number of imported products</th>
<th>Log of import value per product per firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of GDP</td>
<td>1.14***</td>
<td>0.62***</td>
<td>-0.38***</td>
</tr>
<tr>
<td>Log of Distance</td>
<td>-0.79***</td>
<td>-0.13***</td>
<td>-0.72***</td>
</tr>
<tr>
<td>Observations</td>
<td>176</td>
<td>176</td>
<td>176</td>
</tr>
</tbody>
</table>

Source: Data are from the 2000 Linked-Longitudinal Firm Trade Transaction Database (LFTTD). Note: Each column reports the results of a country-specific ordinary least squares regression of the dependent variable noted at the top of each column on the covariates listed on the left. Results for covariates are suppressed. Standard errors are noted below each coefficient. P-values are defined as marginal treatment losses categories.

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Figure 1: Mean value of individual-firm exports (single-region firms, 1992)

Figure 1 from Crozet, M., and P. Koenig. “Structural Gravity Equations with Intensive and Extensive Margins.” Canadian Journal of Economics/Revue canadienne d’économique 43 (2010): 41–62. © John Wiley And Sons Inc. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/fairuse.
Figure 2: Percentage of firms which export (single-region firms, 1992)

Figure 2 from Crozet, M., and P. Koenig. "Structural Gravity Equations with Intensive and Extensive Margins." Canadian Journal of Economics/Revue canadienne d'économique 43 (2010): 41–62. © John Wiley And Sons Inc. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/fairuse.
Table 2: Decomposition of French aggregate industrial exports (34 industries - 159 countries - 1986 to 1992)

<table>
<thead>
<tr>
<th></th>
<th>All firms</th>
<th>Single-region firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Average</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Average</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Number of</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>Number of</td>
<td>(9)</td>
<td>(10)</td>
</tr>
<tr>
<td>shipments</td>
<td>(11)</td>
<td>(12)</td>
</tr>
<tr>
<td>ln (GDP_{ij})</td>
<td>0.461^a</td>
<td>0.417^b</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>ln (Dist_{ij})</td>
<td>-0.325^a</td>
<td>-0.446^a</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Contig_{ij}</td>
<td>-0.064^c</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Colony_{ij}</td>
<td>0.100^a</td>
<td>0.466^a</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>French_{ij}</td>
<td>0.213^a</td>
<td>0.991^a</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.028)</td>
</tr>
<tr>
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<td>0.991^a</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.028)</td>
</tr>
</tbody>
</table>

Note: These are OLS estimates with year and industry dummies. Robust standard errors in parentheses, with ‘^’ denoting significance at the 1%, 5% and 10% level respectively.

Table 2. Decomposing Spatial Frictions (5-digit zip code data)

<table>
<thead>
<tr>
<th></th>
<th>Dist</th>
<th>Dist'</th>
<th>Ownzip</th>
<th>Ownstate</th>
<th>Constant</th>
<th>Adj. R²</th>
<th>N</th>
<th>x_E</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln (value)</td>
<td>0.157</td>
<td>-0.021</td>
<td>0.215</td>
<td>-0.063</td>
<td>-11.900</td>
<td>0.00</td>
<td>1290788</td>
<td>-0.106</td>
</tr>
<tr>
<td>(ijPQP)</td>
<td>(0.096)</td>
<td>(0.001)</td>
<td>(0.026)</td>
<td>(0.006)</td>
<td>(0.026)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>avg. value</td>
<td>0.157</td>
<td>-0.021</td>
<td>0.215</td>
<td>-0.063</td>
<td>-11.900</td>
<td>0.00</td>
<td>1290788</td>
<td>-0.106</td>
</tr>
<tr>
<td>(ijPQP)</td>
<td>(0.096)</td>
<td>(0.001)</td>
<td>(0.026)</td>
<td>(0.006)</td>
<td>(0.026)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>avg. price</td>
<td>-0.032</td>
<td>0.036</td>
<td>-0.115</td>
<td>-0.154</td>
<td>0.021</td>
<td>0.08</td>
<td>1290788</td>
<td>0.419</td>
</tr>
<tr>
<td>(ijPP)</td>
<td>(0.007)</td>
<td>(0.001)</td>
<td>(0.024)</td>
<td>(0.006)</td>
<td>(0.020)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>avg. weight</td>
<td>0.119</td>
<td>-0.058</td>
<td>0.334</td>
<td>0.087</td>
<td>-12.000</td>
<td>0.05</td>
<td>1290788</td>
<td>-0.537</td>
</tr>
<tr>
<td>(ijQQ)</td>
<td>(0.011)</td>
<td>(0.001)</td>
<td>(0.037)</td>
<td>(0.009)</td>
<td>(0.031)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Regression of (log) shipment value and its components from equations (7) and (8) on geographic variables. Dependent variables in left hand column. Coefficients in right-justified rows sum to coefficients in left justified rows.
2. Standard errors in parentheses.
3. x_E is the elasticity of trade with respect to distance, evaluated at the sample mean distance of 523 miles.

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 Courtesy of Russell Hillberry and David Hummels. Used with permission.
2 Helpman, Melitz and Rubenstein (QJE, 2008)

- What does the difference between intensive and extensive margins imply for the estimation of gravity equations?

  - Gravity equations are often used as a tool for measuring trade costs and the determinants of trade costs—we will see an entire lecture on
estimating trade costs later in the course, and gravity equations will
loom large.

• HMR (2008) started wave of thinking about gravity equation estimation
in the presence of extensive/intensive margins.
  – They use aggregate international trade (so this paper doesn’t tech-
     nically belong in a lecture on ‘firm-level trade empirics’) to explore
     implications of a heterogeneous firm model for gravity equation esti-
     mation.
  – The Melitz (2003) model—which you’ll see properly next week—is
     simplified and used as a tool to understand, estimate, and correct for
     biases in gravity equation estimation.

2.1 HMR (2008): Zeros in Trade Data

• HMR start with the observation that there are lots of ‘zeros’ in interna-
tional trade data, even when aggregated up to total bilateral exports.
  – Baldwin and Harrigan (2008) and Johnson (2008) look at this in a
    more disaggregated manner and find (unsurprisingly) far more zeros.

• Zeros are interesting.

• But zeros are also problematic.
  – A typical analysis of trade flows is based on the gravity equation (in
    logs), which can’t incorporate \( X_{ij} = 0 \)
  – Indeed, other models of the gravity equation (Armington, Krugman,
    Eaton-Kortum) don’t have any zeros in them (due to CES and un-
    bounded productivities and finite trade costs).
2.2 A Gravity Model with Zeroes

- HMR work with a multi-country version of Melitz (2003)—similar to Chaney (2008).

- Set-up:
  - Monopolistic competition, CES preferences ($\varepsilon$), one factor of production (unit cost $c_j$), one sector.
- Both variable (iceberg $\tau_{ij}$) and fixed ($f_{ij}$) costs of exporting.
- Heterogeneous firm-level productivities $1/a$ drawn from truncated Pareto, $G(a)$.

- Some firms in $j$ sell in country $i$ iff $a \leq a_{ij}$, where the cutoff productivity ($a_{ij}$) is defined by:

$$\kappa_1 \left( \frac{\tau_{ij} c_j a_{ij}}{P_i} \right)^{1-\varepsilon} Y_i = c_j f_{ij} \quad (1)$$

- HMR (2008) derive a gravity equation, for those observations that are non-zero, of the form:

$$\ln(M_{ij}) = \beta_0 + \alpha_i + \alpha_j - \gamma \ln d_{ij} + w_{ij} + u_{ij} \quad (2)$$

- Where:
  - $M_{ij}$ is imports
  - $d_{ij}$ is distance
  - $w_{ij}$ is the ‘augmented’ part, which is a term accounting for selection.
  - $M_{ij} = 0$ is possible here (even with CES preferences and finite variable trade costs) because it is assumed that each country’s firms have productivities drawn from a bounded (truncated Pareto) distribution.

### 2.3 Two Sources of Bias
- The HMR (2008) theory suggests (and solves) two sources of bias in the typical estimation of gravity equations (which neglects $w_{ij}$).

- **First**: Omitted variable bias due to the presence of $w_{ij}$:
  - In a model with heterogeneous firm productivities and fixed costs of exporting (i.e. a Melitz (2003) model), only highly productive firms will penetrate distant markets.
  - So distance ($d_{ij}$) does two things: it raises the price at which any firm can sell (thus reducing demand along the intensive margin) in, and it changes the productivity (and hence the price and hence the amount sold) of the firms entering, a distant market.
  - This means that $d_{ij}$ is correlated with $w_{ij}$.
  - Therefore, if one aims to estimate $\gamma$ but neglects to control for $w_{ij}$, the estimate of $\gamma$ will be biased (due to OVB).
• The HMR (2008) theory suggests (and solves) two sources of bias in the typical estimation of gravity equations (which neglects $w_{ij}$).

• **Second**: A selection effect induced by only working with non-zero trade flows:
  
  - HMR’s gravity equation, like those before it, can’t be estimated on the observations for which $M_{ij} = 0$.
  
  - The HMR theory tells us that the existence of these ‘zeros’ is not as good as random with respect to $d_{ij}$, so econometrically this ‘selection effect’ needs to be corrected/controlled for.
  
  - Intuitively, the problem is that far away destinations are less likely to be profitable, so the sample of zeros is selected on the basis of $d_{ij}$.
  
  - This calls for a standard Heckman (1979) selection correction.

### 2.4 HMR (2008): Two-step Estimation

1. Estimate probit for zero trade flow or not:
   
   - Include exporter and importer fixed effects, and $d_{ij}$.
   
   - Can proceed with just this, but then identification (in Step 2) is achieved purely off of the normality assumption.
   
   - To ‘strengthen’ identification, need additional variable that enters Probit in step 1, but does not enter Step 2.
   
   - Theory says this should be a variable that affects the fixed cost of exporting, but not the variable cost.
   
   - HMR use Djankov et al (QJE, 2002)’s ‘entry regulation’ index. Also try ‘common religion dummy.’

2. Estimate gravity equation on positive trade flows:
   
   - Include inverse Mills ratio (standard Heckman trick) to control for selection problem (Second source of bias)
   
   - Also include empirical proxy for $w_{ij}$ based on estimate of entry equation in Step 1 (to fix First source of bias).
### Benchmark Gravity and Selection into Trading Relationship

<table>
<thead>
<tr>
<th>Variables</th>
<th>1990's</th>
<th>1990's reduced sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Probit)</td>
<td>(Polynomial)</td>
</tr>
<tr>
<td>Distance</td>
<td>-0.315** (0.021)</td>
<td>-0.300** (0.021)</td>
</tr>
<tr>
<td>Land border</td>
<td>0.00** (0.007)</td>
<td>0.00** (0.007)</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.315** (0.021)</td>
<td>0.310** (0.021)</td>
</tr>
<tr>
<td>Landlock</td>
<td>0.00** (0.007)</td>
<td>0.00** (0.007)</td>
</tr>
<tr>
<td>Legal</td>
<td>0.00** (0.007)</td>
<td>0.00** (0.007)</td>
</tr>
<tr>
<td>Language</td>
<td>0.525** (0.016)</td>
<td>0.515** (0.016)</td>
</tr>
<tr>
<td>Colonial ties</td>
<td>0.525** (0.016)</td>
<td>0.515** (0.016)</td>
</tr>
<tr>
<td>FTA</td>
<td>0.525** (0.016)</td>
<td>0.515** (0.016)</td>
</tr>
<tr>
<td>WTO (none)</td>
<td>0.525** (0.016)</td>
<td>0.515** (0.016)</td>
</tr>
<tr>
<td>WTO (both)</td>
<td>0.525** (0.016)</td>
<td>0.515** (0.016)</td>
</tr>
<tr>
<td>Observations</td>
<td>110,697</td>
<td>110,697</td>
</tr>
</tbody>
</table>

Notes: Exporter, importer, and year fixed effects. Marginal effects at sample means and pseudo R² reported for Probit. Marginal effects at sample means and pseudo R² reported for NLS; robust standard errors (clustering by country pair) elsewhere. 
** Significant at 1%. 
* Significant at 5%. 
+ Significant at 10%.

### Baseline Results

**Image by MIT OpenCourseWare.**

### 3 Crozet and Koenig (CJE, 2010)

- CK (2010) conduct a similar exercise to HMR (2008), but with French firm-level data.
This is attractive—after all, the main point that HMR (2008) is making is that firm-level realities matter for aggregate flows.

• CK’s firm data has exports to foreign countries in it (CK focus only on adjacent countries: Belgium, Switzerland, Germany, Spain and Italy).

3.1 CK (2010): Identification

• But interestingly, CK also know where the firm is in France.

• So they try to separately identify the effects of variable and fixed trade costs by assuming:
  – Variable trade costs are proportional to distance. Since each firm is a different distance from, say, Belgium, there is cross-firm variation here.
  – Fixed trade costs are homogeneous across France for a given export destination. (It costs just as much to figure out how to sell to the Swiss whether your French firm is based in Geneva or Normandy).

3.2 CK (2010): The model and estimation

• The model is deliberately close to Chaney (2008), which is a particular version of the Melitz (2003) model but with (unbounded) Pareto-distributed firm productivities (with shape parameter $\gamma$). We will see this model in detail in the next lecture.

• In Chaney (2008) the elasticity of trade flows with respect to variable trade costs (proxies for by distance here, if we assume $\tau_{ij} = \theta D_{ij}$ where $D =$ distance) can be subdivided into the:
  – Extensive elasticity: $\varepsilon^{EXT}_{D_{ij}} = -\delta [\gamma - (\sigma - 1)]$. CK estimate this by regressing firm-level entry (ie a Probit) on firm-level distance $D_{ij}$ and a firm fixed effect. This is analogous to HMR’s first stage.
  – Intensive elasticity: $\varepsilon^{INT}_{D_{ij}} = -\delta (\sigma - 1)$. CK estimate this by regressing firm-level exports on firm-level distance $D_{ij}$ and a firm fixed effect. This is analogous to HMR’s second stage.

• Recall that $\gamma$ is the Pareto parameter governing firm heterogeneity.

• The above two equations (HMR’s first and second stage) don’t separately identify $\delta$, $\sigma$ and $\gamma$.

  – So to identify the model, CK bring in another equation which is the slope of the firm size (sales) distribution.
In the Chaney (2008) model this will behave as: $X_t = \lambda(c_t)^{-\gamma-(\sigma-1)}$, where $c_t$ is a firm’s marginal cost and $X_t$ is a firm’s total sales.

With an Olley and Pakes (1996) TFP estimate of $1/c_t$, CK estimate $[\gamma - (\sigma - 1)]$ and hence identify the entire system of 3 unknowns.
3.3 CK (2010): Results (each industry separately)

![Image by MIT OpenCourseWare.](image)

**Figure 3:** Comparison of our results for $\sigma$ and $\delta$ with those of Broda and Weinstein (2003)

3.4 CK (2010): Results (do the parameters make sense?)

![Image by MIT OpenCourseWare.](image)

**Figure 4:** The estimated impact of trade barriers and distance on trade margins, by industry

3.5 CK (2010): Results (what do the parameters imply about margins?)

![Image by MIT OpenCourseWare.](image)
• EKK (2009) construct a Melitz (2003)-like model in order to try to capture the key features of French firms’ exporting behavior:
  – Whether to export. (Simple extensive margin).
  – Which countries to export to. (Country-wise extensive margins).
  – How much to export to each country. (Intensive margin).

• They uncover some striking regularities in the firm-wise sales data in (multiple) foreign markets.
  – These ‘power law’ like relationships occur all over the place (Gabaix (ARE survey, 2009)).
  – Most famously, they occur for domestic sales within one market.
  – In that sense, perhaps it’s not surprising that they also occur market by market abroad. (At the heart of power laws is scale invariance.)
4.1 EKK (2009): Stylised Fact 1: Market Entry (averages across countries)

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of exporters</th>
<th>Fraction of exporters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium* (BE)</td>
<td>17,699</td>
<td>0.520</td>
</tr>
<tr>
<td>Germany (DE)</td>
<td>14,579</td>
<td>0.428</td>
</tr>
<tr>
<td>Switzerland (CH)</td>
<td>14,173</td>
<td>0.416</td>
</tr>
<tr>
<td>Italy (IT)</td>
<td>10,643</td>
<td>0.313</td>
</tr>
<tr>
<td>United Kingdom (UK)</td>
<td>9,752</td>
<td>0.287</td>
</tr>
<tr>
<td>Netherlands (NL)</td>
<td>8,294</td>
<td>0.244</td>
</tr>
<tr>
<td>United States (US)</td>
<td>7,608</td>
<td>0.224</td>
</tr>
<tr>
<td><strong>Total Exporters</strong></td>
<td><strong>34,035</strong></td>
<td></td>
</tr>
</tbody>
</table>

* Belgium includes Luxembourg

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4.2 EKK (2009): Stylised Fact 2: Sales Distributions (across all firms)

![Sales Distributions of French Firms](image)

4.3 EKK (2009): Stylised Fact 3: Export Participation and Size in France

![Sales in France and Market Entry](image)

• EKK (2009) therefore add some features to Melitz (2003) in order to bring this model closer to the data.

• Most of these will take the flavor of ‘firm-specific shocks/noise’.
  – The shocks smooths things out, allows for unobserved heterogeneity, and answer the structural econometrician’s question of “where does your regression’s error term come from?”.

• The remaining slides describe some of the features of the EKK model, and how the model matches the data. I include them here just for your interest as they won’t make much sense until you’ve learned the Melitz (2003) model—see the next lecture!

• Shocks:
  – Firm (ie j)-specific productivity draws (in country i): \( z_i(j) \). This is Pareto with parameter \( \theta \).
  – Firm-specific demand draw \( \alpha_n(j) \). The demand they face in market \( n \) is thus: \( X_n(j) = \alpha_n(j)f \left( \frac{X_n}{P_n} \right)^{-(\sigma-1)} \), where \( f \) will be defined shortly.
  – Firm-specific fixed entry costs \( E_{ni}(j) = \varepsilon_n(j)E_{ni}M(f) \), where \( \varepsilon_n(j) \) is the firm-specific ‘fixed exporting cost shock’, \( E_{ni} \) is the fixed exporting term that appears in Melitz (2003) or HMR (2008) (ie constant across firms). And \( M(f) = \frac{1-(1-f)^{1-1/\lambda}}{1-1/\lambda} \), which, following Arkolakis (2008), is a micro-founded ‘marketing’ function that captures how much firms have to pay to ‘access’ \( f \) consumers (this is a choice variable).
  – EKK assume that \( g(\alpha, \varepsilon) \) can take any form, but it needs to be the same across countries \( n \), iid across firms, and within firms independent from the Pareto distribution of \( z \).

• The entry condition is similar to Melitz (2003). Enter if cost \( c_{ni}(j) = \frac{w_{ni}(j)}{z_i(j)} \) satisfies:

\[
c \leq \bar{c}_{ni}(\eta) = \left( \frac{\eta X_n}{\sigma E_{ni}} \right)^{1/(\sigma-1)} \frac{P_n}{\bar{m}} \tag{3}
\]

  – Here \( \eta_n(j) = \frac{\alpha_n(j)}{z_i(j)} \).
  – And \( X_n \) is total sales in \( n \), \( P_n \) is the price index in \( n \), and \( \bar{m} \) is the (constant) markup.
Integrating this over the distribution $g(\eta)$ we know how much entry (measure of firms) there is:

$$J_{ni} = \frac{\kappa_2 \pi_{ni} X_n}{\kappa_1 \sigma E_{ni}}$$  \hspace{1cm} (4)

This therefore agrees well with Fact 1 (normalized entry is linear in $X_n$).

The firm sales (conditional on entry) condition is similar to Arkolakis (2008):

$$X_{ni}(j) = \varepsilon \left[ 1 - \left( \frac{c}{\bar{c}_{ni}(\eta)} \right)^{\lambda(\sigma-1)} \right] \left( \frac{c}{\bar{c}_{ni}(\eta)} \right)^{-\lambda (\sigma-1)} \sigma E_{ni}. \hspace{1cm} (5)$$

There is more work to be done, but one can already see that this will look a lot like a Pareto distribution ($c$ is Pareto, so $c$ to any power is also Pareto) in each market (as in Figure 2).

But the $\left[ 1 - \left( \frac{c}{\bar{c}_{ni}(\eta)} \right)^{\lambda(\sigma-1)} \right]$ will cause the sales distribution to deviate from Pareto in the lower tail (also as in Figure 2).
• The amount of sales in France conditional on entering market \( n \) can be shown to be:

\[
X_{FF}(j)|_n = \frac{\alpha_F(j)}{\eta_n(j)} \left[ 1 - \nu_nF(j)^{\lambda/\theta} \left( \frac{N_nF}{N_{FF}} \right)^{\lambda/\theta} \left( \frac{\eta_n(j)}{\eta_F(j)} \right)^{\lambda} \right] \\
\times \nu_nF(j)^{-1/\theta} \left( \frac{N_nF}{N_{FF}} \right)^{-1/\theta} \frac{\kappa_2}{\kappa_1} \bar{X}_{FF}.
\]

• Since \( N_nF/N_{FF} \) is close to zero (everywhere but in France) the dependence of this on \( N_nF \) is Pareto with slope \(-1/\theta\). As in Figure 3.
Sales in France and Market Entry

- **Sales and Markets Penetrated**
  - Minimum number of markets penetrated vs. average sales in France ($ millions)

- **Sales and # Penetrating Multiple Markets**
  - Number of firms selling to k or more markets vs. average sales in France ($ millions)

- **Sales and # Selling to a Market**
  - Number of firms selling in the market vs. average sales in France ($ millions)

- **Distribution of Sales and Market Entry**
  - Distribution of firms selling in the market vs. average sales in France ($ millions)

Image by MIT OpenCourseWare.