14.581 International Trade
Class notes on 4/8/2013

1 The Armington Model

1.1 Equilibrium

- Labor endowments
  \[ L_i \text{ for } i = 1, \ldots, n \]
- CES utility ⇒ CES price index
  \[ P_j^{1-\sigma} = \sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma} \]
- Bilateral trade flows follow gravity equation:
  \[ X_{ij} = \frac{(w_i \tau_{ij})^{1-\sigma}}{\sum_{l=1}^{n} (w_l \tau_{lj})^{1-\sigma}} w_j L_j \]
- In what follows \( \varepsilon \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \sigma - 1 \) denotes the trade elasticity
- Trade balance
  \[ \sum_{i} X_{ji} = w_j L_j \]

1.2 Welfare Analysis

- Question:
  Consider a foreign shock: \( L_i \rightarrow L_i' \) for \( i \neq j \) and \( \tau_{ij} \rightarrow \tau_{ij}' \) for \( i \neq j \). How do foreign shocks affect real consumption, \( C_j \equiv w_j/P_j \)?

- Shephard’s Lemma implies
  \[ d \ln C_j = d \ln w_j - d \ln P_j = -\sum_{i=1}^{n} \lambda_{ij} (d \ln c_{ij} - d \ln c_{jj}) \]
  with \( c_{ij} \equiv w_i \tau_{ij} \) and \( \lambda_{ij} \equiv X_{ij}/w_j L_j \).
- Gravity implies
  \[ d \ln \lambda_{ij} - d \ln \lambda_{jj} = -\varepsilon (d \ln c_{ij} - d \ln c_{jj}) . \]

\(^1\)The notes are based on lecture slides with inclusion of important insights emphasized during the class.
• Combining these two equations yields

\[ d \ln C_j = \sum_{i=1}^{n} \lambda_{ij} \left( d \ln \lambda_{ij} - d \ln \lambda_{jj} \right) \frac{1}{\varepsilon}. \]

• Noting that \( \sum_i \lambda_{ij} = 1 \implies \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0 \) then

\[ d \ln C_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}. \]

• Integrating the previous expression yields (\( \hat{x} = x' / x \))

\[ \hat{C}_j = \lambda_{jj}^{-1/\varepsilon}. \]

• In general, predicting \( \lambda_{jj} \) requires (computer) work
  – We can use exact hat algebra as in DEK (Lecture #3)
  – Gravity equation + data \( \{\lambda_{ij}, Y_j\} \), and \( \varepsilon \)

• But predicting how bad would it be to shut down trade is easy...
  – In autarky, \( \lambda_{jj} = 1 \). So

\[ C_j^A / C_j = \lambda_{jj}^{1/\varepsilon} \]

  – Thus gains from trade can be computed as

\[ GT_j \equiv 1 - C_j^A / C_j = 1 - \lambda_{jj}^{1/\varepsilon} \]

1.3

1.4 Gains from Trade

• Suppose that we have estimated trade elasticity using gravity equation
  – Central estimate in the literature is \( \varepsilon = 5 \)

• We can then estimate gains from trade:
2 Gravity Models and the Gains from Trade:

ACR (2012)

2.1 Motivation

- New Trade Models
  - Micro-level data have lead to new questions in international trade:
    * How many firms export?
    * How large are exporters?
    * How many products do they export?
  - New models highlight new margins of adjustment:
    * From inter-industry to intra-industry to intra-firm reallocations

- Old question:
  - How large are the gains from trade (GT)?

- ACR’s question:
  - How do new trade models affect the magnitude of GT?

2.2 ACR’s Main Equivalence Result

- ACR focus on gravity models
  - PC: Armington and Eaton & Kortum ’02
  - MC: Krugman ’80 and many variations of Melitz ’03

- Within that class, welfare changes are \((\hat{x} = x' / x)\)
  \[
  \hat{C} = \lambda^{1/\varepsilon}
  \]
• **Two sufficient statistics** for welfare analysis are:
  - Share of domestic expenditure, $\lambda$;
  - Trade elasticity, $\varepsilon$

• **Two views** on ACR’s result:
  - Optimistic: welfare predictions of Armington model are more robust than you thought
  - Pessimistic: within that class of models, micro-level data do not matter

2.3 **Primitive Assumptions**

Preferences and Endowments

• **CES utility**
  - Consumer price index,
    $$P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega,$$

• **One factor of production**: labor
  - $L_i \equiv$ labor endowment in country $i$
  - $w_i \equiv$ wage in country $i$

Technology

• **Linear cost function:**
  $$C_{ij} (\omega, t, q) = \underbrace{q w_i \tau_{ij} \alpha_{ij} (\omega) t^{\frac{1-\sigma}{\sigma}}}_{\text{variable cost}} + \underbrace{w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij} (\omega) m_{ij} (t)}_{\text{fixed cost}},$$

$q$ : quantity,
$\tau_{ij}$ : iceberg transportation cost,
$\alpha_{ij} (\omega)$ : good-specific heterogeneity in variable costs,
$\xi_{ij}$ : fixed cost parameter,
$\phi_{ij} (\omega)$ : good-specific heterogeneity in fixed costs.
$m_{ij} (t)$ : cost for endogenous destination specific technology choice, $t$,
$$t \in \left[ t, t' \right], \ m_{ij}' > 0, \ m_{ij}'' \geq 0$$
• Heterogeneity across goods
  \[ G_j (\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n) \equiv \\{ \omega \in \Omega \mid \alpha_{ij} (\omega) \leq \alpha_i, \phi_{ij} (\omega) \leq \phi_i, \forall i \} \]

Market Structure
• **Perfect competition**
  – Firms can produce any good.
  – No fixed exporting costs.

• **Monopolistic competition**
  – Either firms in \( i \) can pay \( w_i F_i \) for monopoly power over a random good.
  – Or exogenous measure of firms, \( N_i < N \), receive monopoly power.

• Let \( N_i \) be the measure of goods that can be produced in \( i \)
  – Perfect competition: \( N_i = N \)
  – Monopolistic competition: \( N_i < N \)

### 2.4 Macro-Level Restrictions

**Trade is Balanced**
• Bilateral trade flows are
  \[ X_{ij} = \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij} (\omega) \, d\omega \]

• **R1** *For any country \( j \),*
  \[ \sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji} \]
  – Trivial if perfect competition or \( \beta = 0 \).
  – Non trivial if \( \beta > 0 \).

**Profit Share is Constant**
• **R2** *For any country \( j \),*
  \[ \Pi_j / (\sum_{i=1}^n X_{ji}) \text{ is constant} \]
  where \( \Pi_j \): aggregate profits gross of entry costs, \( w_i F_i \), (if any)
Trivial under perfect competition.
Direct from Dixit-Stiglitz preferences in Krugman (1980).
Non-trivial in more general environments.

CES Import Demand System

- Import demand system

\[ (w, N, \tau) \rightarrow X \]

- R3

\[ \varepsilon_{ij}^{ii'} = \frac{\partial \ln (X_{ij}/X_{jj})}{\partial \ln \tau_{i'j}} = \begin{cases} 
\varepsilon < 0 & i = i' \neq j \\
0 & \text{otherwise}
\end{cases} \]

Note: symmetry and separability.

CES Import Demand System

- The trade elasticity \( \varepsilon \) is an upper-level elasticity: it combines
  - \( x_{ij}(\omega) \) (intensive margin)
  - \( \Omega_{ij} \) (extensive margin).

- R3 \( \implies \) complete specialization.

- R1-R3 are not necessarily independent
  - If \( \beta = 0 \) then R3 \( \implies \) R2.

Strong CES Import Demand System (AKA Gravity)

- R3' The IDS satisfies

\[ X_{ij} = \frac{\chi_{ij} \cdot M_i \cdot (w_{i\tau ij})^{\varepsilon} \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot M_{i'} \cdot (w_{i'\tau i'j})^{\varepsilon}} \]

where \( \chi_{ij} \) is independent of \((w, M, \tau)\).

- Same restriction on \( \varepsilon_{ij}^{ii'} \) as R3 but, but additional structural relationships

### 2.5 Welfare results

- State of the world economy:

\[ Z \equiv (L, \tau, \xi) \]

- Foreign shocks: a change from \( Z \) to \( Z' \) with no domestic change.
2.6 Equivalence

- **Proposition 1**: Suppose that R1-R3 hold. Then
  \[ \hat{W}_j = \hat{\lambda}_{jj}^{1/\varepsilon} . \]

- Implication: 2 sufficient statistics for welfare analysis $\hat{\lambda}_{jj}$ and $\varepsilon$

- New margins affect structural interpretation of $\varepsilon$
  - ...and composition of gains from trade (GT)...
  - ... but size of GT is the same.

Gains from Trade Revisited

- Proposition 1 is an *ex-post* result... a simple *ex-ante* result:

- **Corollary 1**: Suppose that R1-R3 hold. Then
  \[ \hat{W}_j^A = \lambda_{jj}^{-1/\varepsilon} . \]

- A stronger ex-ante result for *variable trade costs* under R1-R3':

- **Proposition 2**: Suppose that R1-R3' hold. Then
  \[ \hat{W}_j = \hat{\lambda}_{jj}^{1/\varepsilon} \]

  where
  \[ \hat{\lambda}_{jj} = \left[ \sum_{i=1}^{n} \lambda_{ij} (\hat{w}_i \hat{\tau}_{ij})^{\varepsilon} \right]^{-1} , \]

  and
  \[ \hat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij} \hat{w}_j Y_j (\hat{w}_i \hat{\tau}_{ij})^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j} (\hat{w}_{i'} \hat{\tau}_{i'j})^{\varepsilon}} . \]

- $\varepsilon$ and $\{\lambda_{ij}\}$ are sufficient to predict $\hat{W}_j$ (ex-ante) from $\hat{\tau}_{ij}$, $i \neq j$. 

3.1 Departing from ACR’s (2012) Equivalence Result

- Other Gravity Models:
  - Multiple Sectors
  - Tradable Intermediate Goods
  - Multiple Factors
  - Variable Markups

- Beyond Gravity:
  - PF’s sufficient statistic approach
  - Revealed preference argument (Bernhofen and Brown 2005)
  - More data (Costinot and Donaldson 2011)

3.2 Multiple sectors, GT

- Nested CES: Upper level EoS $\rho$ and lower level EoS $\varepsilon_s$

- Recall gains for Canada of 3.8%. Now gains can be much higher: $\rho = 1$ implies $GT = 17.4\%$

3.3 Tradable intermediates, GT

- Set $\rho = 1$, add tradable intermediates with Input-Output structure

- Labor shares are $1 - \alpha_{j,s}$ and input shares are $\alpha_{j,ks}$ ($\sum_k \alpha_{j,ks} = \alpha_{j,s}$)

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<th>$% GT_{j}^{MS}$</th>
<th>$% GT_{j}^{ID}$</th>
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3.4 Combination of micro and macro features
- In Krugman, free entry ⇒ scale effects associated with total sales
- In Melitz, additional scale effects associated with market size
- In both models, trade may affect entry and fixed costs
- All these effects do not play a role in the one sector model
- With multiple sectors and traded intermediates, these effects come back

3.5 Gains from Trade

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3.6 From GT to trade policy evaluation

- Back to \( \{\lambda_{ij}, Y_j\} \), \( \varepsilon \) and \( \{\hat{r}_{ij}\} \) to get implied \( \hat{\lambda}_{jj} \)

- This is what CGE exercises do

- Contribution of recent quantitative work:
  - Link to theory—“mid-sized models”
  - Model consistent estimation
  - Quantify mechanisms

3.7 Main Lessons from CR (2013)

- Mechanisms that matter for GT:
  - Multiple sectors, tradable intermediates
  - Market structure matters, but in a more subtle way

- Trade policy in gravity models:
  - Good approximation to optimal tariff is \( 1/\varepsilon \approx 20\% \) (related to Gros 87)
  - Large range for which countries gain from tariffs
  - Small effects of tariffs on other countries