14.581 International Trade
Class notes on 3/4/2013

1 Factor Proportion Theory

• The law of comparative advantage establishes the relationship between relative autarky prices and trade flows
  – But where do relative autarky prices come from?

• Factor proportion theory emphasizes **factor endowment differences**

• **Key elements:**
  1. Countries differ in terms of factor abundance [i.e relative factor supply]
  2. Goods differ in terms of factor intensity [i.e relative factor demand]

• Interaction between 1 and 2 will determine differences in relative autarky prices, and in turn, the pattern of trade

• In order to shed light on factor endowments as a source of CA, we will assume that:
  1. Production functions are identical around the world
  2. Households have identical homothetic preferences around the world

• We will first focus on two special models:
  – **Ricardo-Viner** with 2 goods, 1 “mobile” factor (labor) and 2 “im-mobile” factors (sector-specific capital)
  – **Heckscher-Ohlin** with 2 goods and 2 “mobile” factors (labor and capital)

• The second model is often thought of as a long-run version of the first (Neary 1978)
  – In the case of Heckscher-Ohlin, what it is the time horizon such that one can think of total capital as fixed in each country, though freely mobile across sectors?

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1 The notes are based on lecture slides with inclusion of important insights emphasized during the class.
2 Ricardo-Viner Model

2.1 Basic environment

- Consider an economy with:
  - Two goods, \( g = 1, 2 \)
  - Three factors with endowments \( l, k_1, \) and \( k_2 \)

- Output of good \( g \) is given by
  \[
  y_g = f^g (l_g, k_g),
  \]
  where:
  - \( l_g \) is the (endogenous) amount of labor in sector \( g \)
  - \( f^g \) is homogeneous of degree 1 in \( (l_g, k_g) \)

- Comments:
  - \( l \) is a “mobile” factor in the sense that it can be employed in all sectors
  - \( k_1 \) and \( k_2 \) are “immobile” factors in the sense that they can only be employed in one of them
  - Model is isomorphic to DRS model: \( y_g = f^g (l_g) \) with \( f^g_{ll} < 0 \)
  - Payments to specific factors under CRS ≡ profits under DRS

2.2 Equilibrium (I): small open economy

- We denote by:
  - \( p_1 \) and \( p_2 \) the prices of goods 1 and 2
  - \( w, r_1, \) and \( r_2 \) the prices of \( l, k_1, \) and \( k_2 \)

- For now, \( (p_1, p_2) \) is exogenously given: “small open economy”

  - So no need to look at good market clearing

- Profit maximization:
  \[
  p_g f^g_l (l_g, k_g) = w \quad (1)
  
  p_g f^g_k (l_g, k_g) = r_g \quad (2)
  
- Labor market clearing:
  \[
  l = l_1 + l_2 \quad (3)
  
2
2.3 Graphical analysis

- Equations (1) and (3) jointly determine labor allocation and wage

2.4 Comparative statics

- Consider a TOT shock such that $p_1$ increases:
  - $w \nearrow$, $l_1 \nearrow$, and $l_2 \searrow$
  - Condition (2) $\Rightarrow r_1/p_1 \nearrow$ whereas $r_2$ (and a fortiori $r_2/p_1$) $\searrow$

- One can use the same type of arguments to analyze consequences of:
  - Productivity shocks
  - Changes in factor endowments

- In all cases, results are intuitive:
“Dutch disease” (Boom in export sectors, Bids up wages, which leads to a contraction in the other sectors)

Useful political-economy applications (Grossman and Helpman 1994)

• Easy to extend the analysis to more than 2 sectors:
  – Plot labor demand in one sector vs. rest of the economy

2.5 Equilibrium (II): two-country world

• Predictions on the pattern of trade in a two-country world depend on whether differences in factor endowments come from:
  – Differences in the relative supply of specific factors
  – Differences in the relative supply of mobile factors

• Accordingly, any change in factor prices is possible as we move from autarky to free trade (see Feenstra Problem 3.1 p. 98)

3 Two-by-Two Heckscher-Ohlin Model

3.1 Basic environment

• Consider an economy with:
  – Two goods, \( g = 1, 2 \),
  – Two factors with endowments \( l \) and \( k \)

• Output of good \( g \) is given by
  \[
y_g = f^g (l_g, k_g),
\]

where:

– \( l_g, k_g \) are the (endogenous) amounts of labor and capital in sector \( g \)
– \( f^g \) is homogeneous of degree 1 in \( (l_g, k_g) \)

3.2 Back to the dual approach

• \( c_g (w, r) \equiv \) unit cost function in sector \( g \)
  \[
c_g (w, r) = \min_{l, k} \{wl + rk|f^g (l, k) \geq 1\},
\]

where \( w \) and \( r \) the price of labor and capital

• \( a_{fg} (w, r) \equiv \) unit demand for factor \( f \) in the production of good \( g \)
Using the Envelope Theorem, it is easy to check that:

\[
a_{lg}(w, r) = \frac{dc_g(w, r)}{dw} \quad \text{and} \quad a_{kg}(w, r) = \frac{dc_g(w, r)}{dr}
\]

\[A(w, r) \equiv [a_{fg}(w, r)]\] denotes the matrix of total factor requirements.

### 3.3 Equilibrium conditions (I): small open economy

- Like in RV model, we first look at the case of a **“small open economy”**
  - So no need to look at good market clearing

**Profit-maximization:**

\[
p_g \leq wa_{lg}(w, r) + ra_{kg}(w, r) \quad \text{for all } g = 1, 2 \quad (4)
\]

\[
p_g = wa_{lg}(w, r) + ra_{kg}(w, r) \quad \text{if } g \text{ is produced in equilibrium} \quad (5)
\]

**Factor market-clearing:**

\[
l = y_1a_{l1}(w, r) + y_2a_{l2}(w, r) \quad (6)
\]

\[
k = y_1a_{k1}(w, r) + y_2a_{k2}(w, r) \quad (7)
\]

### 3.4 Factor Price Equalization

- **Question:**
  Can trade in goods be a (perfect) substitute for trade in factors?

- First classical result from the HO literature answers by the affirmative

- To establish this result formally, we’ll need the following definition:

  **Definition.** Factor Intensity Reversal (FIR) does not occur if:
  
  \(i\) \(a_{l1}(w, r)/a_{k1}(w, r) > a_{l2}(w, r)/a_{k2}(w, r)\) for all \((w, r)\);
  
  \(ii\) \(a_{l1}(w, r)/a_{k1}(w, r) < a_{l2}(w, r)/a_{k2}(w, r)\)
  
  for all \((w, r)\).

### 3.4.1 Factor Price Insensitivity (FPI)

- **Lemma** If both goods are produced in equilibrium and FIR does not occur, then factor prices \(\omega \equiv (w, r)\) are uniquely determined by good prices \(p \equiv (p_1, p_2)\)

- **Proof:** If both goods are produced in equilibrium, then \(p = A'(\omega)\omega\).
  By Gale and Nikaido (1965), this equation admits a unique solution if \(a_{fg}(\omega) > 0\) for all \(f, g\) and \(\det[A(\omega)] \neq 0\) for all \(\omega\), which is guaranteed by no FIR.

- **Comments:**
– Good prices rather than factor endowments determine factor prices
– In a closed economy, good prices and factor endowments are, of course, related, but not for a small open economy
– All economic intuition can be gained by simply looking at Leontief case
– Proof already suggests that “dimensionality” will be an issue for FIR

Factor Price Insensitivity (FPI): graphical analysis

• Link between no FIR and FPI can be seen graphically:

![Diagram showing FPI](image)

• If iso-cost curves cross more than once, then FIR must occur

3.4.2 Factor Price Equalization (FPE) Theorem

• The previous lemma directly implies (Samuelson 1949) that:

• **FPE Theorem** If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices

• Comments:

  – Trade in goods can be a “perfect substitute” for trade in factors
  – Countries with different factor endowments can sustain same factor prices through different allocation of factors across sectors
  – Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries...
  – For next results, we’ll maintain assumption that both goods are produced in equilibrium, but won’t need free trade and same technology
3.5 Stolper-Samuelson (1941) Theorem

- **Stolper-Samuelson Theorem** An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduced the real return to the other factor

- **Proof:** W.l.o.g. suppose that (i) \( a_{i1} (\omega) / a_{i1} (\omega) > a_{i2} (\omega) / a_{i2} (\omega) \) and (ii) \( \hat{p}_2 > \hat{p}_1 \). Differentiating the zero-profit condition (5), we get
  \[
  \hat{p}_g = \theta_{ig} \hat{w} + (1 - \theta_{ig}) \hat{r},
  \]
  where \( \hat{x} = d \ln x \) and \( \theta_{ig} \equiv w a_{ig} (\omega) / c_g (\omega) \). Equation (8) implies
  \[
  \hat{w} \geq \hat{p}_1, \hat{p}_2 \geq \hat{r} \text{ or } \hat{r} \geq \hat{p}_1, \hat{p}_2 \geq \hat{w}
  \]
  By (i), \( \theta_{i2} < \theta_{i1} \). So (i) requires \( \hat{r} > \hat{w} \). Combining the previous inequalities, we get
  \[
  \hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}
  \]

- **Comments:**
  - Previous “hat” algebra is often referred to “Jones’ (1965) algebra”
  - The chain of inequalities \( \hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w} \) is referred as a “magnification effect”
  - SS predict both winners and losers from change in relative prices
  - Like FPI and FPE, SS entirely comes from zero-profit condition (+ no joint production)
  - Like FPI and FPE, sharpness of the result hinges on “dimensionality”
  - In the empirical literature, people often talk about “Stolper-Samuelson effects” whenever looking at changes in relative factor prices (though changes in relative good prices are rarely observed)

Stolper-Samuelson (1941) Theorem: graphical analysis

![Graphical analysis](image-url)
Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontief case:

- In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies

3.6 Rybczynski (1965) Theorem

- Previous results have focused on the implication of zero profit condition, Equation (5), for factor prices

- Now turn our attention to the implication of factor market clearing, Equations (6) and (7), for factor allocation

- **Rybczynski Theorem** An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry

- **Proof**: W.l.o.g. suppose that (i) \( a_{l1} (\omega) / a_{k1} (\omega) > a_{l2} (\omega) / a_{k2} (\omega) \) and (ii) \( \hat{k} > \hat{l} \). Differentiating factor market clearing conditions (6) and (7), we get

  \[
  \hat{l} = \lambda_{l1} \hat{y}_1 + (1 - \lambda_{l1}) \hat{y}_2 \tag{9}
  \]

  \[
  \hat{k} = \lambda_{k1} \hat{y}_1 + (1 - \lambda_{k1}) \hat{y}_2 \tag{10}
  \]

  where \( \lambda_{l1} = a_{l1} (\omega) y_1 / \lambda_{l1} \) and \( \lambda_{k1} = a_{k1} (\omega) y_1 / \lambda_{k1} \). Equations (8) implies

  \( \hat{y}_1 \geq \hat{l}, \hat{k} \geq \hat{y}_2 \) or \( \hat{y}_2 \geq \hat{l}, \hat{k} \geq \hat{y}_1 \)

  By (i), \( \lambda_{k1} < \lambda_{l1} \). So (ii) requires \( \hat{y}_2 > \hat{y}_1 \). Combining the previous inequalities, we get

  \( \hat{y}_2 > \hat{k} > \hat{l} > \hat{y}_1 \)

- Like for FPI and FPE Theorems:

  - \((p_1, p_2)\) is exogenously given \(\Rightarrow\) factor prices and factor requirements are not affected by changes factor endowments

  - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy

- Like for SS Theorem, we have a “magnification effect”

- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on “dimensionality”
Rybczynski (1965) Theorem: graphical analysis (I)

- Since good prices are fixed, it is as if we were in Leontieff case

\[ y_2 = a_{ll} y_1 + a_{l2} y_2 \]

\[ k = a_{kl} y_1 + a_{k2} y_2 \]

Rybczynski (1965) Theorem: graphical analysis (II)

- Rybczynski effect can also be illustrated using relative factor supply and relative factor demand:

\[ \frac{r}{l} \]

\[ \frac{K}{L} \]

- Cross-sectoral reallocations are at the core of HO predictions:
  - For relative factor prices to remain constant, aggregate relative demand must go up, which requires expansion capital intensive sector