Today’s Plan

1. The Simplest Gravity Model: Armington
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The Armington Model

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The Armington Model: Equilibrium

- Labor endowments
  \[ L_i \text{ for } i = 1, \ldots, n \]

- CES utility \(\Rightarrow\) CES price index
  \[ P_j^{1-\sigma} = \sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma} \]

- Bilateral trade flows follow gravity equation:
  \[ X_{ij} = \frac{(w_i \tau_{ij})^{1-\sigma}}{\sum_{l=1}^{n} (w_l \tau_{lj})^{1-\sigma}} w_j L_j \]

- In what follows \( \varepsilon \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \sigma - 1 \) denotes the trade elasticity

- Trade balance
  \[ \sum_{i} X_{ji} = w_j L_j \]
The Armington Model: Welfare Analysis

- **Question:**
  Consider a foreign shock: $L_i \rightarrow L'_i$ for $i \neq j$ and $\tau_{ij} \rightarrow \tau'_{ij}$ for $i \neq j$. How do foreign shocks affect real consumption, $C_j \equiv w_j / P_j$?

- Shephard’s Lemma implies

  $$d \ln C_j = d \ln w_j - d \ln P_j = - \sum_{i=1}^{n} \lambda_{ij} \left( d \ln c_{ij} - d \ln c_{jj} \right)$$

  with $c_{ij} \equiv w_i \tau_{ij}$ and $\lambda_{ij} \equiv X_{ij} / w_j L_j$.

- Gravity implies

  $$d \ln \lambda_{ij} - d \ln \lambda_{jj} = -\varepsilon \left( d \ln c_{ij} - d \ln c_{jj} \right).$$
Combining these two equations yields

\[ d \ln C_j = \sum_{i=1}^{n} \lambda_{ij} \left( d \ln \lambda_{ij} - d \ln \lambda_{jj} \right) \frac{1}{\varepsilon}. \]

Noting that \( \sum_i \lambda_{ij} = 1 \implies \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0 \) then

\[ d \ln C_j = -\frac{d \ln \lambda_{jj}}{\varepsilon}. \]

Integrating the previous expression yields \( (x' = x'/x) \)

\[ \hat{C}_j = \hat{\lambda}_{jj}^{-1/\varepsilon}. \]
The Armington Model: Welfare Analysis

- In general, predicting $\hat{\lambda}_{jj}$ requires (computer) work
  - We can use exact hat algebra as in DEK (Lecture #3)
  - Gravity equation + data $\{\lambda_{ij}, Y_j\}$, and $\varepsilon$
- But predicting how bad would it be to shut down trade is easy...
  - In autarky, $\lambda_{jj} = 1$. So

\[
C_j^A / C_j = \lambda_{jj}^{1/(\sigma-1)}
\]

- Thus gains from trade can be computed as

\[
GT_j \equiv 1 - C_j^A / C_j = 1 - \lambda_{jj}^{1/\varepsilon}
\]
The Armington Model: Gains from Trade

- Suppose that we have estimated trade elasticity using gravity equation
  - Central estimate in the literature is $\varepsilon = 5$
- We can then estimate gains from trade:

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New Trade Models

Micro-level data have lead to **new questions** in international trade:
- How many firms export?
- How large are exporters?
- How many products do they export?

New models highlight **new margins** of adjustment:
- From inter-industry to intra-industry to intra-firm reallocations

Old question:
- How large are the gains from trade (GT)?

ACR’s question:
- How do new trade models affect the magnitude of GT?
ACR’s Main Equivalence Result

- ACR focus on gravity models
  - PC: Armington and Eaton & Kortum ’02
  - MC: Krugman ’80 and many variations of Melitz ’03
- Within that class, welfare changes are \((\hat{x} = x'/x)\)
  \[ \hat{C} = \hat{\lambda}^{1/\varepsilon} \]

- **Two sufficient statistics** for welfare analysis are:
  - Share of domestic expenditure, \(\lambda\);
  - Trade elasticity, \(\varepsilon\)
- **Two views** on ACR’s result:
  - Optimistic: welfare predictions of Armington model are more robust than you thought
  - Pessimistic: within that class of models, micro-level data do not matter
**Primitive Assumptions**

Preferences and Endowments

- **CES utility**
  - Consumer price index,
  
  \[ P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega, \]

- **One factor of production:** labor
  - \( L_i \equiv \) labor endowment in country \( i \)
  - \( w_i \equiv \) wage in country \( i \)
**Linear cost function:**

\[ C_{ij}(\omega, t, q) = q w_i \tau_{ij} \alpha_{ij}(\omega) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^\beta \zeta_{ij} \phi_{ij}(\omega) m_{ij}(t), \]

- \( q \): quantity,
- \( \tau_{ij} \): iceberg transportation cost,
- \( \alpha_{ij}(\omega) \): good-specific heterogeneity in variable costs,
- \( \zeta_{ij} \): fixed cost parameter,
- \( \phi_{ij}(\omega) \): good-specific heterogeneity in fixed costs.
Linear cost function:

\[ C_{ij} (\omega, t, q) = q \omega_i \tau_{ij} \alpha_{ij} (\omega) t^{\frac{1}{1-\sigma}} + w_{ij}^{1-\beta} \xi_{ij} \phi_{ij} (\omega) m_{ij} (t) \]

\( m_{ij} (t) \): cost for endogenous destination specific technology choice, \( t \),

\[ t \in [t, \bar{t}] , \ m'_{ij} > 0 , \ m''_{ij} \geq 0 \]
Primitive Assumptions

Technology

- **Linear cost function:**

\[ C_{ij}(\omega, t, q) = qw_i \tau_{ij} \alpha_{ij}(\omega) t^{1-\sigma} + w_i^{1-\beta} w_j^\beta \xi_{ij} \phi_{ij}(\omega) m_{ij}(t) \]

- **Heterogeneity across goods**

\[ G_j(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n) \equiv \{ \omega \in \Omega | \alpha_{ij}(\omega) \leq \alpha_i, \phi_{ij}(\omega) \leq \phi_i, \forall i \} \]
**Primitive Assumptions**

**Market Structure**

- **Perfect competition**
  - Firms can produce any good.
  - No fixed exporting costs.

- **Monopolistic competition**
  - Either firms in $i$ can pay $w_i F_i$ for monopoly power over a random good.
  - Or exogenous measure of firms, $\overline{N}_i < \overline{N}$, receive monopoly power.

- Let $N_i$ be the measure of goods that can be produced in $i$
  - Perfect competition: $N_i = \overline{N}$
  - Monopolistic competition: $N_i < \overline{N}$
Macro-Level Restrictions
Trade is Balanced

- Bilateral trade flows are
  \[ X_{ij} = \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij}(\omega) \, d\omega \]

- **R1 For any country** \( j \),
  \[ \sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji} \]

  - Trivial if perfect competition or \( \beta = 0 \).
  - Non trivial if \( \beta > 0 \).
Macro-Level Restrictions
Profit Share is Constant

- **R2** For any country $j$,

\[ \frac{\Pi_j}{\left( \sum_{i=1}^{n} X_{ji} \right)} \text{ is constant} \]

where $\Pi_j$ : aggregate profits gross of entry costs, $w_jF_j$, (if any)

- Trivial under perfect competition.
- Direct from Dixit-Stiglitz preferences in Krugman (1980).
- Non-trivial in more general environments.
Macro-Level Restriction

CES Import Demand System

- **Import demand system**

  \[(w, N, \tau) \rightarrow X]\n
- **R3**

  \[\varepsilon_{ij}' \equiv \frac{\partial \ln (X_{ij} / X_{jj})}{\partial \ln \tau_{i'j}} = \begin{cases} 
  \varepsilon < 0 & i = i' \neq j \\
  0 & \text{otherwise} 
  \end{cases}\]

- **Note:** symmetry and separability.
The trade elasticity $\varepsilon$ is an upper-level elasticity: it combines

- $x_{ij}(\omega)$ (intensive margin)
- $\Omega_{ij}$ (extensive margin).

R3 $\implies$ complete specialization.

R1-R3 are not necessarily independent

- If $\beta = 0$ then R3 $\implies$ R2.
Macro-Level Restriction
Strong CES Import Demand System (AKA Gravity)

- **R3’** The IDS satisfies

\[
X_{ij} = \frac{\chi_{ij} \cdot M_i \cdot (w_i \tau_{ij})^\varepsilon \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot M_{i'} \cdot (w_{i'} \tau_{i'j})^\varepsilon}
\]

where \(\chi_{ij}\) is independent of \((w, M, \tau)\).

- Same restriction on \(\varepsilon_{ij}'\) as R3 but, but additional structural relationships
Welfare results

- State of the world economy:
  \[ Z \equiv (L, \tau, \xi) \]

- Foreign shocks: a change from \( Z \) to \( Z' \) with no domestic change.
Equivalence (I)

- **Proposition 1:** *Suppose that R1-R3 hold. Then*

  \[ \hat{W}_j = \hat{\lambda}_{jj}^{1/\varepsilon}. \]

- Implication: 2 sufficient statistics for welfare analysis \( \hat{\lambda}_{jj} \) and \( \varepsilon \)

- New margins affect structural interpretation of \( \varepsilon \)
  - ...and composition of gains from trade (GT)...
  - ... but size of GT is the same.
Proposition 1 is an *ex-post* result... a simple *ex-ante* result:

**Corollary 1:** Suppose that R1-R3 hold. Then

\[ \hat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}. \]
A stronger ex-ante result for **variable trade costs** under R1-R3':

**Proposition 2:** Suppose that R1-R3' hold. Then

\[
\hat{W}_j = \hat{\lambda}_{jj}^{1/\varepsilon}
\]

where

\[
\hat{\lambda}_{jj} = \left[ \sum_{i=1}^{n} \lambda_{ij} \left( \hat{w}_i \hat{\tau}_{ij} \right)^\varepsilon \right]^{-1},
\]

and

\[
\hat{w}_i = \sum_{j=1}^{n} \lambda_{ij} \hat{w}_j Y_j \left( \hat{w}_i \hat{\tau}_{ij} \right)^\varepsilon
\]

\(\varepsilon\) and \(\{\lambda_{ij}\}\) are sufficient to predict \(W_j\) (ex-ante) from \(\hat{\tau}_{ij}, i = j\).
Taking Stock

ACR consider models featuring:

- (i) Dixit-Stiglitz preferences;
- (ii) one factor of production;
- (iii) linear cost functions; and
- (iv) perfect or monopolistic competition;

with three macro-level restrictions:

- (i) trade is balanced;
- (ii) aggregate profits are a constant share of aggregate revenues; and
- (iii) a CES import demand system.

Equivalence for ex-post welfare changes and GT

- under R3’ equivalence carries to ex-ante welfare changes
3. Beyond ACR’s (2012) Equivalence Result:
   CR (2013)
Departing from ACR’s (2012) Equivalence Result

**Other Gravity Models:**
- Multiple Sectors
- Tradable Intermediate Goods
- Multiple Factors
- Variable Markups

**Beyond Gravity:**
- PF’s sufficient statistic approach
- Revealed preference argument (Bernhofen and Brown 2005)
- More data (Costinot and Donaldson 2011)
1. Add multiple sectors

2. Add traded intermediates
Multiple sectors, GT

- Nested CES: Upper level EoS $\rho$ and lower level EoS $\varepsilon_s$

- Recall gains for Canada of 3.8%. Now gains can be much higher: $\rho = 1$ implies $GT = 17.4\%$
- Set $\rho = 1$, add tradable intermediates with Input-Output structure
- Labor shares are $1 - \alpha_{j,s}$ and input shares are $\alpha_{j,ks}$ ($\sum_k \alpha_{j,ks} = \alpha_{j,s}$)
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Combination of micro and macro features

- In Krugman, free entry $\Rightarrow$ scale effects associated with total sales
- In Melitz, additional scale effects associated with market size
- In both models, trade may affect entry and fixed costs
- All these effects do not play a role in the one sector model
- With multiple sectors and traded intermediates, these effects come back
## Gains from Trade

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From GT to trade policy evaluation

- Back to $\{\lambda_{ij}, Y_j\}$, $\varepsilon$ and $\{\hat{\tau}_{ij}\}$ to get implied $\hat{\lambda}_{jj}$

- This is what CGE exercises do

- Contribution of recent quantitative work:
  - Link to theory—“mid-sized models”
  - Model consistent estimation
  - Quantify mechanisms
Main Lessons from CR (2013)

- **Mechanisms that matter for GT:**
  - Multiple sectors, tradable intermediates
  - Market structure matters, but in a more subtle way

- **Trade policy in gravity models:**
  - Good approximation to optimal tariff is $1/\epsilon \approx 20\%$ (related to Gros 87)
  - Large range for which countries gain from tariffs
  - Small effects of tariffs on other countries
For Future Research

- Treatment of capital goods
- Modeling of trade imbalances
- Fit of model
- Relation with micro studies
- Relation with other non-gravity approaches