Putting Ricardo to Work

- Ricardian model has long been perceived as a useful pedagogic tool, with little empirical content:
  - Great to explain undergrads why there are gains from trade
  - But grad students should study richer models (Feenstra’s graduate textbook has a total of 3 pages on the Ricardian model!)
- Eaton and Kortum (2002) have lead to “Ricardian revival”
  - Same basic idea as in Wilson (1980): Who cares about the pattern of trade for counterfactual analysis?
  - But more structure: Small number of parameters, so well-suited for quantitative work
- **Goals of this lecture:**
  1. Present EK model
  2. Discuss estimation of its key parameter
  3. Introduce tools for welfare and counterfactual analysis
Basic Assumptions

- $N$ countries, $i = 1, \ldots, N$
- Continuum of goods $u \in [0, 1]$
- Preferences are CES with elasticity of substitution $\sigma$:
  \[
  U_i = \left( \int_0^1 q_i(u)^{(\sigma-1)/\sigma} \, du \right)^{\sigma/(\sigma-1)}
  \]
- One factor of production (labor)
- There may also be intermediate goods (more on that later)
- $c_i \equiv$ unit cost of the “common input” used in production of all goods
  - Without intermediate goods, $c_i$ is equal to wage $w_i$ in country $i$
Basic Assumptions (Cont.)

- Constant returns to scale:
  - $Z_i(u)$ denotes productivity of (any) firm producing $u$ in country $i$
  - $Z_i(u)$ is drawn independently (across goods and countries) from a Fréchet distribution:
    \[
    \Pr(Z_i \leq z) = F_i(z) = e^{-T_i z^{-\theta}},
    \]
    with $\theta > \sigma - 1$ (important restriction, see below)
  - Since goods are symmetric except for productivity, we can forget about index $u$ and keep track of goods through $Z \equiv (Z_1, ..., Z_N)$.

- Trade is subject to iceberg costs $d_{ni} \geq 1$
  - $d_{ni}$ units need to be shipped from $i$ so that 1 unit makes it to $n$

- All markets are perfectly competitive
Let $P_{ni}(Z) \equiv c_id_{ni}/Z_i$ be the unit cost at which country $i$ can serve a good $Z$ to country $n$ and let $G_{ni}(p) \equiv \Pr(P_{ni}(Z) \leq p)$. Then:

$$G_{ni}(p) = \Pr(Z_i \geq c_id_{ni}/p) = 1 - F_i(c_id_{ni}/p)$$

Let $P_n(Z) \equiv \min\{P_{n1}(Z), ..., P_{nN}(Z)\}$ and let $G_n(p) \equiv \Pr(P_n(Z) \leq p)$ be the price distribution in country $n$. Then:

$$G_n(p) = 1 - \exp[-\Phi_n p^\theta]$$

where

$$\Phi_n \equiv \sum_{i=1}^{N} T_i(c_id_{ni})^{-\theta}$$
To show this, note that (suppressing notation \( Z \) from here onwards)

\[
\Pr(P_n \leq p) = 1 - \Pi_i \Pr(P_{ni} \geq p) \\
= 1 - \Pi_i [1 - G_{ni}(p)]
\]

Using

\[
G_{ni}(p) = 1 - F_i(c_id_{ni}/p)
\]

then

\[
1 - \Pi_i [1 - G_{ni}(p)] = 1 - \Pi_i F_i(c_id_{ni}/p) \\
= 1 - \Pi_i e^{-T_i(c_id_{ni})^\theta} p^\theta
\]

\[
= 1 - e^{-\Phi np^\theta}
\]
Consider a particular good. Country $n$ buys the good from country $i$ if $i = \arg \min \{p_{n1}, \ldots, p_{nN}\}$. The probability of this event is simply country $i$'s contribution to country $n$'s price parameter $\Phi_n$,

$$\pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}$$

To show this, note that

$$\pi_{ni} = \Pr \left( P_{ni} \leq \min_{s \neq i} P_{ns} \right)$$

If $P_{ni} = p$, then the probability that country $i$ is the least cost supplier to country $n$ is equal to the probability that $P_{ns} \geq p$ for all $s \neq i$.
The previous probability is equal to

\[ \prod_{s \neq i} \Pr(P_{ns} \geq p) = \prod_{s \neq i} [1 - G_{ns}(p)] = e^{-\Phi_{n}^{-i}p^\theta} \]

where

\[ \Phi_{n}^{-i} = \sum_{s \neq i} T_i (c_i d_{ni})^{-\theta} \]

Now we integrate over this for all possible \( p \)'s times the density \( dG_{ni}(p) \) to obtain

\[
\int_{0}^{\infty} e^{-\Phi_{n}^{-i}p^\theta} T_i (c_i d_{ni})^{-\theta} \theta p^{\theta-1} e^{-T_i(c_i d_{ni})^{-\theta} p^\theta} dp
\]

\[
= \left( \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_{n}} \right) \int_{0}^{\infty} \theta \Phi_{n} e^{-\Phi_{n}p^\theta} p^{\theta-1} dp
\]

\[
= \pi_{ni} \int_{0}^{\infty} dG_{n}(p) dp = \pi_{ni}
\]
The price of a good that country \( n \) actually buys from any country \( i \) also has the distribution \( G_n(p) \).

To show this, note that if country \( n \) buys a good from country \( i \) it means that \( i \) is the least cost supplier. If the price at which country \( i \) sells this good in country \( n \) is \( q \), then the probability that \( i \) is the least cost supplier is

\[
\Pi_{s \neq i} \Pr(P_{ni} \geq q) = \Pi_{s \neq i} [1 - G_{ns}(q)] = e^{-\Phi_n^{-i} q^\theta}
\]

The joint probability that country \( i \) has a unit cost \( q \) of delivering the good to country \( n \) and is the least cost supplier of that good in country \( n \) is then

\[
e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q)
\]
Integrating this probability $e^{-\Phi_n^{-1} q^\theta} dG_{ni}(q)$ over all prices $q \leq p$ and using $G_{ni}(q) = 1 - e^{-T_i(c_i d_{ni})^{-\theta} p^\theta}$ then

$$
\int_0^p e^{-\Phi_n^{-1} q^\theta} dG_{ni}(q) = \int_0^p e^{-\Phi_n^{-1} q^\theta} \frac{T_i(c_i d_{ni})^{-\theta} q^\theta - 1}{\Phi_n} e^{-T_i(c_i d_{ni})^{-\theta} p^\theta} dq
$$

$$
= \left(\frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}\right) \int_0^p e^{-\Phi_n q^\theta} \theta \Phi_n q^{\theta - 1} dq
$$

$$
= \pi_{ni} G_n(p)
$$

Given that $\pi_{ni} \equiv$ probability that for any particular good country $i$ is the least cost supplier in $n$, then conditional distribution of the price charged by $i$ in $n$ for the goods that $i$ actually sells in $n$ is

$$
\frac{1}{\pi_{ni}} \int_0^p e^{-\Phi_n^{-1} q^\theta} dG_{ni}(q) = G_n(p)
$$
In Eaton and Kortum (2002):

1. All the adjustment is at the extensive margin: countries that are more distant, have higher costs, or lower $T'$s, simply sell a smaller range of goods, but the average price charged is the same.

2. The share of spending by country $n$ on goods from country $i$ is the same as the probability $\pi_{ni}$ calculated above.

We will establish a similar property in models of monopolistic competition with Pareto distributions of firm-level productivity.
The exact price index for a CES utility with elasticity of substitution \( \sigma < 1 + \theta \), defined as

\[
p_n = \left( \int_0^1 p_n(u)^{1-\sigma} \, du \right)^{1/(1-\sigma)},
\]

is given by

\[
p_n = \gamma \Phi_n^{-1/\theta}
\]

where

\[
\gamma = \left[ \Gamma \left( \frac{1-\sigma}{\theta} + 1 \right) \right]^{1/(1-\sigma)}
\]

where \( \Gamma \) is the Gamma function, i.e. \( \Gamma(a) \equiv \int_0^\infty x^{a-1} e^{-x} \, dx \).
To show this, note that
\[
p_n^{1-\sigma} = \int_0^1 p_n(u)^{1-\sigma} du = \int_0^\infty p_1^{1-\sigma} dG_n(p) = \int_0^\infty p_1^{1-\sigma} \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta} dp.
\]

Defining \(x = \Phi_n p^\theta\), then \(dx = \Phi_n \theta p^{\theta-1}\), \(p_1^{1-\sigma} = (x/\Phi_n)^{(1-\sigma)/\theta}\), and
\[
p_n^{1-\sigma} = \int_0^\infty \left(\frac{x}{\Phi_n}\right)^{(1-\sigma)/\theta} e^{-x} dx
= \Phi_n^{-(1-\sigma)/\theta} \int_0^\infty x^{(1-\sigma)/\theta} e^{-x} dx
= \Phi_n^{-(1-\sigma)/\theta} \Gamma \left(\frac{1-\sigma}{\theta} + 1\right)
\]

This implies \(p_n = \gamma \Phi_n^{-1/\theta}\) with \(\frac{1-\sigma}{\theta} + 1 > 0\) or \(\sigma - 1 < \theta\) for gamma function to be well defined.
Equilibrium

- Let $X_{ni}$ be total spending in country $n$ on goods from country $i$
- Let $X_n \equiv \sum_i X_{ni}$ be country $n$’s total spending
- We know that $X_{ni}/X_n = \pi_{ni}$, so

$$X_{ni} = \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} X_n$$ \hfill (*)

- Suppose that there are no intermediate goods so that $c_i = w_i$.
- In equilibrium, total income in country $i$ must be equal to total spending on goods from country $i$ so

$$w_i L_i = \sum_n X_{ni}$$

- Trade balance further requires $X_n = w_n L_n$ so that

$$w_i L_i = \sum_n \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_j T_j (w_j d_{nj})^{-\theta}} w_n L_n$$
This provides system of $N - 1$ independent equations (Walras’ Law) that can be solved for wages $(w_1, \ldots, w_N)$ up to a choice of numeraire.

Everything is as if countries were exchanging labor.

- Frechet distributions imply that labor demands are iso-elastic.
- Armington model leads to similar eq. conditions under assumption that each country is exogenously specialized in a differentiated good.
- In the Armington model, the labor demand elasticity simply coincides with elasticity of substitution $\sigma$.

Under frictionless trade ($d_{ni} = 1$ for all $n, i$) previous system implies

$$w_i^{1+\theta} = \frac{T_i}{L_i} \frac{\sum_n w_n L_n}{\sum_j T_j w_j^{-\theta}}$$

and hence

$$\frac{w_i}{w_j} = \left(\frac{T_i / L_i}{T_j / L_j}\right)^{1/(1+\theta)}$$
The Gravity Equation

- Letting $Y_i = \sum_n X_{ni}$ be country $i$’s total sales, then

$$Y_i = \sum_n \frac{T_i (c_i d_{ni})^{-\theta} X_n}{\Phi_n} = T_i c_i^{-\theta} \Omega_i^{-\theta}$$

where

$$\Omega_i^{-\theta} \equiv \sum_n \frac{d_{ni}^{-\theta} X_n}{\Phi_n}$$

- Solving $T_i c_i^{-\theta}$ from $Y_i = T_i c_i^{-\theta} \Omega_i^{-\theta}$ and plugging into (*) we get

$$X_{ni} = \frac{X_n Y_i d_{ni}^{-\theta} \Omega_i^{\theta}}{\Phi_n}$$

- Using $p_n = \gamma \Phi_n^{-1/\theta}$ we can then get

$$X_{ni} = \gamma^{-\theta} X_n Y_i d_{ni}^{-\theta} (p_n \Omega_i)^{\theta}$$

- This is the **Gravity Equation**, with bilateral resistance $d_{ni}$ and multilateral resistance terms $p_n$ (inward) and $\Omega_i$ (outward).
From (*) we also get that country $i$’s share in country $n$’s expenditures normalized by its own share is

$$S_{ni} \equiv \frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\Phi_i d_{ni}^{-\theta}}{\Phi_n} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$$

This shows the importance of trade costs and comparative advantage in determining trade volumes. Note that if there are no trade barriers (i.e., frictionless trade), then $S_{ni} = 1$.

Letting $B_{ni} \equiv \left(\frac{X_{ni}}{X_{ii}} \cdot \frac{X_{in}}{X_{nn}}\right)^{1/2}$ then

$$B_{ni} = (S_{ni} S_{in})^{1/2} = \left(d_{ni}^{-\theta} d_{in}^{-\theta}\right)^{1/2}$$

Under symmetric trade costs (i.e., $d_{ni} = d_{in}$) then $B_{ni}^{-1/\theta} = d_{ni}$ can be used as a measure of trade costs.
We can also see how $B_{ni}$ varies with physical distance between $n$ and $i$: 

![Graph showing the relationship between normalized import share and distance (in miles) between countries $n$ and $i$. The graph indicates a negative correlation, with normalized import share decreasing as distance increases.](Image by MIT OpenCourseWare.)
How to Estimate the Trade Elasticity?

- As we will see the trade elasticity $\theta$ is the key structural parameter for welfare and counterfactual analysis in EK model.
- Cannot estimate $\theta$ directly from $B_{ni} = d_{ni}^{-\theta}$ because distance is not an empirical counterpart of $d_{ni}$ in the model.
  - Negative relationship in Figure 1 could come from strong effect of distance on $d_{ni}$ or from mild CA (high $\theta$).
- Consider again the equation

$$S_{ni} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$$

- If we had data on $d_{ni}$, we could run a regression of $\ln S_{ni}$ on $\ln d_{ni}$ with importer and exporter dummies to recover $\theta$.
  - But how do we get $d_{ni}$?
EK use price data to measure \( p_i d_{ni} / p_n \):

- They use retail prices in 19 OECD countries for 50 manufactured products from the UNICP 1990 benchmark study.
- They interpret these data as a sample of the prices \( p_i(j) \) of individual goods in the model.
- They note that for goods that \( n \) imports from \( i \) we should have \( p_n(j) / p_i(j) = d_{ni} \), whereas goods that \( n \) doesn’t import from \( i \) can have \( p_n(j) / p_i(j) \leq d_{ni} \).
- Since every country in the sample does import manufactured goods from every other, then \( \max_j \{ p_n(j) / p_i(j) \} \) should be equal to \( d_{ni} \).
- To deal with measurement error, they actually use the second highest \( p_n(j) / p_i(j) \) as a measure of \( d_{ni} \).
Let $r_{ni}(j) \equiv \ln p_n(j) - \ln p_i(j)$. They calculate $\ln(p_n/p_i)$ as the mean across $j$ of $r_{ni}(j)$. Then they measure $\ln(p_i d_{ni}/p_n)$ by

$$D_{ni} = \max 2_j \left\{ r_{ni}(j) \right\} / \sum_j r_{ni}(j) / 50$$

Given $S_{ni} = \left( p_i d_{ni} / p_n \right)^{-\theta}$ they estimate $\theta$ from $\ln(S_{ni}) = -\theta D_{ni}$.

Method of moments: $\theta = 8.28$. OLS with zero intercept: $\theta = 8.03$. 

© Econometric Society. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/fairuse.
Simonovska and Waugh (2011) argue that EK’s procedure suffers from upward bias:
- Since EK are only considering 50 goods, maximum price gap may still be strictly lower than trade cost
- If we underestimate trade costs, we overestimate trade elasticity
- Simulation based method of moments leads to a $\theta$ closer to 4.

An alternative approach is to use tariffs (Caliendo and Parro, 2011). If $d_{ni} = t_{ni} \tau_{ni}$ where $t_{ni}$ is one plus the ad-valorem tariff (they actually do this for each 2 digit industry) and $\tau_{ni}$ is assumed to be symmetric, then

$$\frac{X_{ni}X_{ij}X_{jn}}{X_{nj}X_{ji}X_{in}} = \left( \frac{d_{ni}d_{ij}d_{jn}}{d_{nj}d_{ji}d_{in}} \right)^{-\theta} = \left( \frac{t_{ni}t_{ij}t_{jn}}{t_{nj}t_{ji}t_{in}} \right)^{-\theta}$$

They can then run an OLS regression and recover $\theta$. Their preferred specification leads to an estimate of 8.22
Consider again the case where \( c_i = w_i \)

From (*), we know that

\[
\pi_{nn} = \frac{X_{nn}}{X_n} = \frac{T_n w_n^{-\theta}}{\Phi_n}
\]

We also know that \( p_n = \gamma \Phi_n^{-1/\theta} \), so

\[
\omega_n \equiv w_n / p_n = \gamma^{-1} T_n^{1/\theta} \pi_{nn}^{-1/\theta}.
\]

Under autarky we have \( \omega_n^A = \gamma^{-1} T_n^{1/\theta} \), hence the gains from trade are given by

\[
GT_n \equiv \omega_n / \omega_n^A = \pi_{nn}^{-1/\theta}
\]

Trade elasticity \( \theta \) and share of expenditure on domestic goods \( \pi_{nn} \) are sufficient statistics to compute GT.
A typical value for $\pi_{nn}$ (manufacturing) is 0.7. With $\theta = 5$ this implies $GT_n = 0.7^{-1/5} = 1.074$ or 7.4% gains. Belgium has $\pi_{nn} = 0.2$, so its gains are $GT_n = 0.2^{-1/5} = 1.38$ or 38%.

One can also use the previous approach to measure the welfare gains associated with any foreign shock, not just moving to autarky:

$$\omega_n'/\omega_n = (\pi_{nn}'/\pi_{nn})^{-1/\theta}$$

For more general counterfactual scenarios, however, one needs to know both $\pi_{nn}'$ and $\pi_{nn}$. 
Adding an Input-Output Loop

- Imagine that intermediate goods are used to produce a composite good with a CES production function with elasticity $\sigma > 1$. This composite good can be either consumed or used to produce intermediate goods (input-output loop).

- Each intermediate good is produced from labor and the composite good with a Cobb-Douglas technology with labor share $\beta$. We can then write $c_i = w_i^\beta p_i^{1-\beta}$. 
The analysis above implies
\[ \pi_{nn} = \gamma^{-\theta} T_n \left( \frac{c_n}{p_n} \right)^{-\theta} \]

and hence
\[ c_n = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n \]

Using \( c_n = w_n^\beta p_n^{1-\beta} \) this implies
\[ w_n^\beta p_n^{1-\beta} = \gamma^{-1} T_n^{-1/\theta} \pi_{nn}^{-1/\theta} p_n \]

so
\[ w_n / p_n = \gamma^{-1/\beta} T_n^{-1/\theta \beta} \pi_{nn}^{-1/\theta \beta} \]

The gains from trade are now
\[ \omega_n / \omega_n^A = \pi_{nn}^{-1/\theta \beta} \]

Standard value for \( \beta \) is 1/2 (Alvarez and Lucas, 2007). For \( \pi_{nn} = 0.7 \) and \( \theta = 5 \) this implies \( GT_n = 0.7^{-2/5} = 1.15 \) or 15% gains.
Adding Non-Tradables

- Assume now that the composite good cannot be consumed directly.
- Instead, it can either be used to produce intermediates (as above) or to produce a consumption good (together with labor).
- The production function for the consumption good is Cobb-Douglas with labor share $\alpha$.
- This consumption good is assumed to be non-tradable.
The price index computed above is now \( p_{gn} \), but we care about 
\( \omega_n \equiv w_n / p_{fn} \), where 

\[ p_{fn} = w_n \alpha p_{gn} \]

This implies that 

\[ \omega_n = \frac{w_n}{w_n \alpha p_{gn}} = (w_n / p_{gn})^{1-\alpha} \]

Thus, the gains from trade are now 

\[ \frac{\omega_n}{\omega_n^A} = \pi_{nn}^{-\eta} / \theta \]

where 
\[ \eta \equiv \frac{1 - \alpha}{\beta} \]

Alvarez and Lucas argue that \( \alpha = 0.75 \) (share of labor in services). Thus, for \( \pi_{nn} = 0.7 \), \( \theta = 5 \) and \( \beta = 0.5 \), this implies 
\( GT_n = 0.7^{-1/10} = 1.036 \) or 3.6% gains
Go back to the simple EK model above ($\alpha = 0$, $\beta = 1$). We have

$$X_{ni} = \gamma^{-\theta} T_i(w_i d_{ni})^{-\theta} p_n^\theta X_n$$

$$p_n^{-\theta} = \gamma^{-\theta} \sum_{i=1}^{N} T_i(w_i d_{ni})^{-\theta}$$

$$\sum_{n} X_{ni} = w_i L_i$$

As we have already established, this leads to a system of non-linear equations to solve for wages,

$$w_i L_i = \sum_{n} \frac{T_i(w_i d_{ni})^{-\theta}}{\sum_{k} T_k (w_k d_{nk})^{-\theta}} w_n L_n.$$
Comparative statics (Dekle, Eaton and Kortum, 2008)

- Consider a shock to labor endowments, trade costs, or productivity. One could compute the original equilibrium, the new equilibrium and compute the changes in endogenous variables.

- But there is a simpler way that uses only information for observables in the initial equilibrium, trade shares and GDP; the trade elasticity, \( \theta \); and the exogenous shocks. First solve for changes in wages by solving

\[
\hat{w}_i \hat{L}_i Y_i = \sum_n \frac{\pi_{ni} \hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}} \hat{w}_n \hat{L}_n Y_n
\]

and then get changes in trade shares from

\[
\hat{\pi}_{ni} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}}.
\]

- From here, one can compute welfare changes by using the formula above, namely \( \hat{\omega}_n = (\hat{\pi}_{nn})^{-1/\theta} \).
Comparative statics (Dekle, Eaton and Kortum, 2008)

To show this, note that trade shares are

$$\pi_{ni} = \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_k T_k (w_k d_{nk})^{-\theta}} \quad \text{and} \quad \pi'_{ni} = \frac{T'_i (w'_i d'_{ni})^{-\theta}}{\sum_k T'_k (w'_k d'_{nk})^{-\theta}}.$$  

Letting $\hat{x} \equiv x'/x$, then we have

$$\hat{\pi}_{ni} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta} / \sum_j T_j (w_j d_{nj})^{-\theta}} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta} \ T_k (w_k d_{nk})^{-\theta} / \sum_j T_j (w_j d_{nj})^{-\theta}} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}}.$$
On the other hand, for equilibrium we have

$$w'_i L'_i = \sum_n \pi'_n w'_n L'_n = \sum_n \hat{\pi}_n \pi_n w'_n L'_n$$

Letting $Y_n \equiv w_n L_n$ and using the result above for $\hat{\pi}_n$ we get

$$\hat{w}_i \hat{L}_i Y_i = \sum_n \frac{\pi_n \hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}} \hat{w}_n \hat{L}_n Y_n$$

This forms a system of $N$ equations in $N$ unknowns, $\hat{w}_i$, from which we can get $\hat{w}_i$ as a function of shocks and initial observables (establishing some numeraire). Here $\pi_n$ and $Y_i$ are data and we know $\hat{d}_{ni}$, $\hat{T}_i$, $\hat{L}_i$, as well as $\theta$. 

14.581 (Week 2) Ricardian Theory (I) Spring 2013 32 / 34
Comparative statics (Dekle, Eaton and Kortum, 2008)

- To compute the implications for welfare of a foreign shock, simply impose that $\hat{L}_n = \hat{T}_n = 1$, solve the system above to get $\hat{w}_i$ and get the implied $\hat{\pi}_{nn}$ through

$$\hat{\pi}_{ni} = \frac{\hat{T}_i (\hat{w}_i \hat{d}_{ni})^{-\theta}}{\sum_k \pi_{nk} \hat{T}_k (\hat{w}_k \hat{d}_{nk})^{-\theta}}.$$ 

and use the formula to get

$$\hat{\omega}_n = \hat{\pi}_{nn}^{-1/\theta}.$$

- Of course, if it is not the case that $\hat{L}_n = \hat{T}_n = 1$, then one can still use this approach, since it is easy to show that in autarky one has $w_n / p_n = \gamma^{-1} T_n^{1/\theta}$, hence in general

$$\hat{\omega}_n = \left(\hat{T}_n\right)^{1/\theta} \hat{\pi}_{nn}^{-1/\theta}.$$
Extensions of EK

  - Bertrand competition ⇒ variable markups at the firm-level
  - Measured productivity varies across firms ⇒ one can use firm-level data to calibrate model

- **Multiple Sectors**: Costinot, Donaldson, and Komunjer (2012)
  - \( T^k_i \equiv \) fundamental productivity in country \( i \) and sector \( k \)
  - One can use EK’s machinery to study pattern of trade, not just volumes

- **Non-homothetic preferences**: Fieler (2011)
  - Rich and poor countries have different expenditure shares
  - Combined with differences in \( \theta^k \) across sectors \( k \), one can explain pattern of North-North, North-South, and South-South trade