Labor Economics Problem Set 2

A. Home economics

1. Children are often said to complicate their parents’ labor supply decisions.
   
   (a) Use a model of labor supply with home production to show how single mothers’ labor supply decisions might be affected by child care costs.

   (b) Compare and contrast the likely labor supply consequences of these policies:
       i. A lump-sum child allowance that’s independent of earnings
       ii. A child care tax credit that cuts out at the poverty line
       iii. Making child care expenses tax deductible

   Your analyses should consider: effects on participation, effects on hours for women likely to work anyway, variation in effects as a function of wage rates.

   (c) Identify channels through which state-subsidized out-of-home care (through programs like Head Start) might affect children. What does the evidence suggest?

B. CES production

Consider the constant elasticity of substitution (CES) production function

$$F(K, L) = (\alpha K^{\rho} + (1 - \alpha)L^{\rho})^{\frac{1}{\rho}}$$

with $\rho \in (-\infty, 1]$ and $\alpha \in (0, 1)$.

1. Derive the conditional factor demands ($K^c$ and $L^c$) as well as the cost function for CES production.

2. The elasticity of substitution is defined as

$$\frac{\partial \log (L^c/K^c)}{\partial \log (w^c/r)}$$

Derive the elasticity of substitution for CES production.

3. CES production nests important special cases

   (a) What happens to CES production when $\rho = 1$? What’s this called?

   (b) Derive the limit of the CES production function as $\rho \to 0$. What do we call this?

   (c) Derive the limit of the CES production function as $\rho \to -\infty$. What do we call this?

C. Fast-food labor demand

Suppose that $N$ small fast food establishments produce according to $f(L) = \log L$, with fixed capital. The product price is fixed at 1. Aggregate labor supply is given by $w^e$ for positive $\epsilon$.

1. Assume firms are price-takers in the factor market (in other words, the labor market is competitive). Derive an individual firm’s demand curve. Use this to derive the aggregate demand curve.

2. Derive the competitive equilibrium wage, $w_c$, and aggregate employment level, $N \cdot L_c$, as a function of the labor supply elasticity and the number of firms.

3. Now, suppose that the ghost of Ray Kroc (“that’s Kroc with a K!”) buys these $N$ establishments and operates them as one firm. Assume that the retail fast food market remains competitive. However, monopsonist Kroc has market power in his factor market – in fact he is the sole employer of this type of labor. Derive the new equilibrium wage, $w_m$, and the aggregate employment level, $N \cdot L_m$, and compare these to wages and employment under competition.
4. Now suppose Larry Summers gives up his K-School river view and returns to Washington. Explain how a wise and beneficent policy-maker can use the minimum wage to generate the competitive employment level as an equilibrium outcome in spite of Kroc’s market power. What must Larry know to accomplish this feat?

D. Theoretical immigration effects

In recent years, many skilled immigrants have come to Silicon Valley to work as code warriors in the software industry. Assume there are two types of programmers: natives (Type 1) and immigrants (Type 2). Initially, assume there are \( n_1 \) native programmers in the labor force and no immigrants. Assume also that the aggregate labor supply function of native programmers is \( n_1 S_1(w_1) \), where \( S_1(w_1) \) is the labor supply function of an individual programmer. The demand function for native labor is \( D(w) \), with \( D'(w) < 0 \).

1. Suppose that immigrant programmers have labor supply functions identical to natives’ supply functions and that employers treat immigrant and native programmers as perfect substitutes. Use graphs to show the effect of an influx of \( n_2 \) immigrants on native programmers. Compare this with the effect of immigration if immigrant labor supply is perfectly inelastic.

2. By totally differentiating equilibrium conditions, derive comparative statics formulas for the effect of \( dn_2 \) on native employment in the second two scenarios from part 1. Which effect is larger? More generally, what economic parameters are likely to govern immigration effects on native employment?

E. Empirical immigration effects

1. Try to replicate the results in Table III of Borjas (2003).

2. Explore the robustness of these findings to the possible presence of group-specific trends.

F. Human capital

1. Suppose that potential log earnings for a worker with \( s \) years of schooling are given by

\[
g_i(s) = \alpha + \rho_1 s + \rho_2 s^2
\]

and that potential schooling values \([S_{0i}, S_{1i}]\) indexed against a Bernoulli instrument, \( Z_i \), determine actual schooling according to

\[
S_i = S_{0i} + (S_{1i} - S_{0i})Z_i
\]

Show that under these assumptions, the Wald estmand using \( Z_i \) to instrument \( S_i \) equals the average derivative \( E(\omega_i g_i'(S_i)) \) where

\[
\omega_i = \frac{S_{0i} + S_{1i}}{2}, \quad \frac{S_{1i} - S_{0i}}{E[S_{1i} - S_{0i}]}
\]

In other words, IV captures a weighted average return to schooling over a range of schooling values and for a set of workers determined by the normalized first stage, \( \omega_i \).

2. This weighted averaging property of IV is sometimes said to explain why IV estimates tend to exceed the corresponding OLS estimates. This is what Lang (1993) dubbed a “discount rate bias” explanation for relatively high IV estimates. Why is this reasonably called a sort of “bias”? After all, OLS is also a weighted average with weights that tend to peak at median schooling (for details, see Appendix A in Angrist and Krueger, 1999).

3. Use the AK-91 data set to test the discount rate bias hypothesis by comparing IV estimates using different instruments or computing IV estimates for different groups.