1. Inefficient Learning-by-Doing

As some of you noted in class, talent discovery is not the only setting where a worker’s inability to accept a negative wage may lead to inefficiency. This question asks you to consider the market for careers where profession-specific learning-by-doing is crucial to productivity (e.g., lawyers, financiers...or academic economists!) and certain long-term contracts (that is, indentured servitude) are not enforceable.

Suppose workers in a given profession live for two periods and are employed via a spot labor market. The profession is competitive, so neither firms nor workers may earn rents over their outside option and both take market wages as given. Industry wages are given by \( w_t = p h_t - \phi \), where \( y \) is the output of the worker in period \( t \), \( p \) is the market price of output, and \( \phi \) is a fixed per-worker cost. Suppose worker output is exogenously higher in the second period: \( y_2 > y_1 > 0 \) and their lifetime utility is given by \( u(c_1) + u(c_2) \) where \( c_t \) denotes consumption in period \( t \) and \( u''(c) < 0 < u'(c) \). Young workers may borrow (with zero interest) an amount \( b < L \) against their future wages, where \( L \) reflects possible liquidity constraints.

(a) Young workers are indifferent between entering the profession and going outside the industry, where they would earn a constant wage \( w_0 \). Write an equilibrium condition reflecting this, and use it to derive an expression for prices and wages when \( L = \infty \) (i.e. workers are unconstrained). Under what conditions must a worker pay to enter the profession (that is, \( w^*_1 < 0 \))?

(b) Derive and interpret an expression for optimal borrowing \( b^* \). Suppose \( L = b^* \), so that workers are just able to borrow their desired amount. Derive an expression for how prices and wages respond to a marginal decrease in \( L \). Discuss.

(c) Let us now endogenize worker productivity. Suppose \( y_t = \theta_t e_t \), where \( e_t \) denotes a worker’s effort in period \( t \) and \( \theta_2 > \theta_1 > 0 \). To simplify the analysis assume worker utility is given by

\[
V(c_1, c_2, e_1, e_2) = \sum_{t=1}^{2} \alpha \ln(e_t) + (1 - \alpha) \ln(1 - e_t)
\]

for \( \alpha \in (0, 1) \). Again suppose workers are unconstrained in their borrowing. Write an expression for how workers choose effort in the two periods and solve for effort and consumption in terms of output prices. How are prices determined?

(d) Again suppose \( L = b^* \) so that workers are just able to make their borrowing requirements. How do prices respond to a marginal decrease in \( L \)? How does effort respond? Discuss.
2. A Brief History of Gravity

Trade economists were quite aware of the empirical deficiencies of Heckscher-Ohlin-style models well before Eaton and Kortum’s 2002 seminal *Econometrica* paper. For example McCallum (1995)\(^1\) finds trade between Canadian provinces to be more than 22 times higher than trade between Canada and the U.S., despite a high degree of integration between these two economies. This is strong motivation for the kinds of “iceberg” costs that gravity models take seriously.

McCallum’s estimating equation is of the form

\[
\ln X_{ij} = \pi + \alpha \ln GDP_i + \beta \ln GDP_j + \gamma \ln \text{dist}_{ij} + \delta D_{ij} + \epsilon_{ij}
\]

where \(X_{ij}\) denotes the value of exports from region \(i\) (either a U.S. state or a Canadian province) to region \(j\), \(\text{dist}_{ij}\) is the distance between region \(i\) and \(j\), and \(D_{ij}\) equals 1 if both \(i\) and \(j\) are Canadian provinces, zero otherwise (the coefficient \(\delta\) measures the importance of the U.S.-Canada border to trade that McCallum reports). Our goal is to motivate this sort of regression by a simple gravity model.

For simplicity, suppose each region \(i\) specializes in a single good, the total supply of which is fixed. Preferences for goods are homothetic and identical across regions, given by

\[
U_j = \left( \sum_i q_{ij}^{\rho} \right)^{1/\rho}
\]

where \(q_{ij}\) denotes the quantity of imports to country \(j\) from country \(i\). The price of region \(i\)’s good to region \(j\) is given by \(p_{ij} = \tau_{ij}p_i\) where \(\tau_{ij}\) is an exogenous transport cost. The value of exports from \(i\) to \(j\) can thus be written \(X_{ij} = p_{ij}q_{ij}\). Assume all countries are small, and so take prices as given.

(a) Write the constrained problem country \(j\) solves in deciding how much to import from each country

(b) Derive the nominal demand for region \(i\)’s goods by region \(j\), and use this to write an expression for \(X_{ij}\).

(c) Use market clearing to show there exists an equilibrium with symmetric trading costs \((\tau_{ij} = \tau_{ji} \forall i, j)\) where

\[
P_j = \left( \sum_i \left( \frac{\tau_{ij}}{P_i} \right)^{\rho/(\rho-1)} \frac{Y_i}{Y_W} \right)^{(\rho-1)/\rho}
\]

where \(Y_i\) is the nominal income of region \(i\), \(P_i\) is the price index of region \(i\), and \(Y_W = \sum_i Y_i\). Use this to derive a simple gravity equation for bilateral trade flows \(X_{ij}\) in terms of \(Y_i, Y_j, P_i, P_j, Y_W, \) and \(\tau_{ij}\).

(d) Suppose \(\rho \in (0, 1)\). What is the effect of increased trading costs on trade flows between country \(i\) and \(j\)? What is the effect of increased \(P_i\) or \(P_j\) holding \(\tau_{ij}\) constant? Explain.

(e) Suppose we model trading costs as

\[
\tau_{ij} = (1 + b_{ij}) \text{dist}_{ij}^c
\]

where \(b_{ij}\) equals the tariff equivalent of the border between country \(i\) and \(j\) and \(c\) is a scalar. Use your gravity equation to write down an estimating equation similar to McCallum’s. What issues with McCallum’s empirical strategy does this exercise raise?

A labor market institution that we did not discuss in class but which is widespread throughout the developing and developed world is mandatory labor “standards” that place constraints on the type of contracts that can be struck (for example, indentured servitude). While injunctions on debt bondage or, say, child labor are relatively non-controversial, there are many other labor standards for which the economic or moral case may be less unambiguous; for example: maximum hours limitations, minimum safety requirements, or rules against certain types of treatment by employers (e.g. sexual harassment). The neoclassical economist in you naturally chafes at these regulations. If a worker is willing to accept somewhat less safe working conditions in exchange for a higher rate of pay, why shouldn’t she be allowed to? A simple laissez faire argument says that market transactions among consenting adults that don’t produce negative externalities on others should be permissible.

In this question you’ll explore one aspect of this discussion. Consider a market for sexual harassment. Firms produce output using labor $n$, where $n$ is the number of workers. Production occurs according to $Y = f(n)$, where $f(n)$ is strictly increasing and strictly concave. Employers get perverse gratification $\theta > 0$ (measured in units of output) from each worker they are allowed to harass. Consider two cases. In the first, sexual harassment is illegal. The wage of each worker is given by $w_N^I$ and profits are

$$\pi(n_N) = f(n_N) - n_Nw_N^I$$

In the second, sexually harassment is permitted, and the wages of harassed and non-harassed workers are $w_H^I$ and $w_N^A$, respectively. The firm’s payoff is then

$$\pi(n_N, n_H) = f(n_N + n_H) + n_H\theta - n_Nw_N^A - n_Hw_H^A$$

Assume a unit mass of workers each with labor supply function $s(w)$, where $s'(w) > 0$. Write the monetized cost of harassment to worker $i$ as $c(i)$; workers are indexed such that the individual with the highest disutility of harassment is identified by $i = 0$ and the worker with the lowest disutility is $i = 1$. Assume $c(·)$ is strictly increasing and continuously differentiable, and let $\phi(k)$ be its inverse. Assume $c(0) > \theta > c(1)$.

(a) Derive an expression for equilibrium labor supply in the regimes where harassment is illegal $(B)$ and where it is legal $(A)$. Draw a graph depicting the labor market equilibrium in the $A$ regime.

(b) Can you sign $w_N^I \equiv w_N^B - w_N^A$ or is its sign indeterminate?

(c) Are any workers better off in the $A$ regime than they would be in the $B$ regime?

(d) Are there any non-harassed workers in the $A$ regime who are worse off than they would be in the $B$ regime?

(e) Are there any harassed workers in the $A$ regime who are worse off than they would be in the $B$ regime?

(f) We noted above that an individual worker’s agreeing to tolerate sexual harassment in exchange for payment does not generate negative externalities. Interpret your answers to (d) and (e) in light of this observation.

(g) Congress wants to write a law that legalizes sexual harassment while guaranteeing that the law is Pareto improving. Assuming all of the parameters of the model above are known, can this law be written, and if so how? Illustrate diagrammatically.