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"Fréchet-ing up" Roy (Hsieh et al. 2013)

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A Convergence in Occupational Distributions

- In 1960, 94% of doctors and lawyers were white men; by 2008, 62%
  - Similar “evening-out” for many high-skill occupations (e.g. Blau, 1998)

- Changes in innate talent distribution unlikely to explain this trend
  - Recent correction in workers not pursuing comparative advantage?

- Canonical Roy (1951) model explains “micro” job choice, but not “macro” implications on aggregate productivity
  - Usually don’t model the kinds of group-specific frictions in job choice/human capital production that are needed for this story
  - Technical change may also favor some groups (e.g. the pill)

- Hsieh, Hurst, Jones, and Klenow (2013) fix this with a modified Roy
  - Follow Eaton and Kortum (2002) in assuming talent is Fréchet
  - Produces a tractable expression for the group-occupation distribution
  - Can estimate (sorta...) friction parameters with Census data
Human Capital Accumulation and Job Choice

- Utility of individuals in group $g$ given consumption $c$ and leisure $1 - s$:
  \[ U = c^\beta (1 - s) \]

- Work in occupation $i$ when old; when young earn human capital by
  \[ h(e, s) = \bar{h}_i g s^\phi_i e^\eta \]
  Return to schooling $\phi_i$ is occupation-specific; $e$ is other expenditure

- Two frictions: discrimination in human capital accumulation (e.g. segregated schools) and in the labor market (e.g. Becker, 1957)
  \[ c = w\varepsilon (1 - \tau^w_{ig}) h(e, s) - e (1 + \tau^h_{ig}) \]
  where $w$ is the wage and $\varepsilon$ is an idiosyncratic “talent” draw

- Individual’s indirect utility:
  \[ U(\tau^w, \tau^h, \bar{h}, w, \varepsilon) = \max_{e, s} \left( w\varepsilon (1 - \tau^w_{ig}) \bar{h}_i g s^\phi_i e^\eta - e (1 + \tau^h_{ig}) \right)^\beta (1 - s) \]
Optimal Human Capital Investment

- FOC:
  \[ s_i^* = \left( 1 + \frac{1 - \eta}{\beta \phi_i} \right)^{-1}, \quad e_{ig}^* = \left( \frac{1 - \tau_{ig}^w}{1 + \tau_{ig}^h} \eta w_i \varepsilon h_{ig} s_i^* \phi_i \right)^{1/(1-\eta)} \]

- Schooling time increasing in returns \( \phi_i \), not affected by frictions
  - Frictions (and wages) have same effect on return/cost of time; other expenditures \( e_{ig} \) needed to make distortions “observable” in data

- Plugging back in:
  \[ U_{ig} = \eta^\eta (1 - \eta)^{1-\eta} \cdot \frac{w_i \varepsilon_i s_i^\phi_i (1 - s_i)^{1-\eta} / \beta}{(1 + \tau_{ig}^h) \eta / (\bar{h}_{ig} (1 - \tau_{ig}^w))} \]

- \( \bar{h}_{ig} \) won’t be separately identified from the “gross tax rate” if \( \varepsilon_i \) is unobserved. HHJK normalize \( \bar{h}_{ig} = 1 \)
“Fréchet-ing it Up”

- Assume across occupations $i = 1, \ldots, N$,

$$F_g(\epsilon_1, \ldots, \epsilon_N) = \exp \left( - \left( \sum_{i=1}^{N} T_{ig} \epsilon_i^{-\theta} \right)^{1-\rho} \right)$$

- Here $\rho \in (0,1)$ governs within-person skill correlation and $\theta$ the overall dispersion of skills

- HHJK let $T_{ig}$ vary across sex (“brawny jobs”) but not across race

- Individuals choose the occupation with the highest $U_{ig}$; by our choice of $F_g$, the fraction of people in group $g$ working in occupation $i$ is

$$p_{ig} = \frac{\tilde{w}_{ig}^\theta}{\sum_{s=1}^{N} \tilde{w}_{sg}^\theta}, \text{ where } \tilde{w}_{ig} \equiv \frac{T_{ig}^{1/\theta} w_i s_i^{\phi_i} (1 - s_i)^{(1-\eta)/\beta}}{(1 + \tau_{ig}^h)^{\eta} / (1 - \tau_{ig}^w)}$$
Fréchet-ing Implications

- Sorting depends on $\tilde{w}_{ig}$, the overall “reward” an individual from group $g$ with mean talent obtains from working in occupation $i$.
  - Depends on $T_{ig}$, the “post friction” wage, and time in school.
  - Note: no misallocation if frictions are the same across occupations.

- Average quality of workers in each occupation for a given group:

$$E[h_i \varepsilon_i | g] = \gamma \left( s_i \left( \frac{w_i (1 - \tau_{ig}^w)}{(1 + \tau_{ig}^h)} \right)^\eta \left( \frac{T_{ig}}{p_{ig}} \right)^{1/\theta} \right)^{1/(1-\eta)}$$

where $\gamma = \eta \Gamma(1 - \frac{1}{\theta(1-\rho)} \frac{1}{1-\eta})$ is an integration constant.

- Quality inversely related to the share of the group in the occupation.
  - Only the most talented female lawyers chose that profession in 1960.
Fréchet-ing Implications (cont.)

- Average wages in occupation $i$ for group $g$:

$$\bar{w}_{ig} = (1 - \tau_{ig}^w)w_i E[h_i\epsilon_i|g] = C(1 - s_i)^{-1/\beta} \left( \sum_{s=1}^{N} \tilde{w}_{sg}^\theta \right)^{1/(\theta(1-\eta))}$$

- The wage gap between any two groups is the same across occupations:
  - Higher earnings from lower frictions offset by less productive entrants
  - Feature highly specific to Fréchet choice: key to identification

- Putting together the pieces, we get a estimable model for occupations:

$$\frac{p_{ig}}{p_{i,wm}} = \frac{T_{ig}}{T_{i,wm}} \left( \frac{\tau_{ig}}{\tau_{i,wm}} \right)^{-\theta} \left( \frac{\bar{w}_{ig}}{\bar{w}_{i,wm}} \right)^{-\theta(1-\eta)}$$

Where $\tau_{ig} \equiv (1 + \tau_{ig}^h)^\eta / (1 - \tau_{ig}^w)$ is the overall friction and $wm$ indicates the reference group (i.e. white men)

- Relative mean talent arguably equal to one for many occupations
Closing the Model

- Assuming a representative firm with CES production over occupations:

\[
Y = \left( \sum_{i=1}^{N} \left( A_i H_i \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
\]

where, for group size \( q_g \),

\[
H_i = \sum_{g=1}^{G} q_g p_{ig} \cdot E[h_i \varepsilon_i | g]
\]

- A competitive equilibrium is one where all individuals choose occupations to maximize utility, firms maximize profits, and wages clear the labor market.

- Key prediction: frictions reduce productivity and average wages due to underinvestment in human capital and misallocation of talent across occupations.
Earnings, occupations, and wages of white/black men/women from U.S. Census (1960-2000) and ACS ('06-'08)

Fréchet smell-test #1: occupation-specific wage gap should be uncorrelated with occupation frictions

Fréchet smell-test #2: changes in wage gap should be uncorrelated with changes in occupational propensities
Estimating Frictions

- Recall the model implies

\[
\hat{\tau}_{ig} \equiv \frac{\tau_{ig}}{\tau_{i,wm}} \left( \frac{T_{i,wm}}{T_{i,g}} \right)^{1/\theta} = \left( \frac{p_{ig}}{p_{i,wm}} \right)^{-1/\theta} \left( \frac{\bar{w}_g}{\bar{w}_{wm}} \right)^{-(1-\eta)}
\]

- Interpretation: if a group is either underrepresented in an occupation or faces a large average wage gap, RHS will be large
  - Model rationalizes this by low mean talent and/or high frictions
- Goal: estimate \( \theta \) and \( \eta \) off distributional assumptions, plug in observed \( p_{ig} \) and \( \bar{w}_g \) to back out LHS
- Fréchet implies within occupation-group wages are such that

\[
\frac{\text{Variance}}{\text{Mean}^2} = \frac{\Gamma(1-2/(\theta(1-\rho)(1-\eta)))}{\Gamma(1-1/(\theta(1-\rho)(1-\eta)))} - 1
\]

HHJK use this to (somewhat opaquely) get \( \theta(1-\eta) \approx 3.44 \)
Estimating Frictions (cont.)

- \( \eta \) affects only the level of \( \hat{\tau}_{ig} \); HHJK pick \( \eta = 0.25 \)

Figure 3: Estimated Barriers (\( \hat{\tau}_{ig} \)) for White Women

Figure 4: Estimated Barriers (\( \hat{\tau}_{ig} \)) for Black Men

- Substantial barriers in some occupations; e.g. women lawyers in 1960 received only 1/3 their marginal products
- All frictions fell 1960-2008, with some of the biggest gains concentrated in high-skill sectors

Courtesy of Chang-Tai Hsieh, Erik Hurst, Charles I. Jones, and Peter J. Klenow. Used with permission.
Solving the Model: Female LFP

- HHJK calibrate with $\sigma = 3$ and $\beta = 0.693$ (Mincerian RtS), back out remaining parameters from simple moments in the data (and show some robustness to calibration)
- Set $T_{ig} = \tau_{i,wm} = 1$ in baseline (everything comes from $\tau^w$ or $\tau^h$)
- Use estimates to look at some interesting decompositions and counterfactuals. Here’s a sample of some that I found interesting

Table 8: Female Participation Rates

<table>
<thead>
<tr>
<th></th>
<th>$\tau^h$ case</th>
<th>$\tau^w$ case</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Women's LF participation</strong></td>
<td>1960 = 0.329</td>
<td>2008 = 0.692</td>
</tr>
<tr>
<td><strong>Change, 1960 – 2008</strong></td>
<td>0.364</td>
<td></td>
</tr>
<tr>
<td>Due to changing $\tau$'s</td>
<td>0.235</td>
<td>0.262</td>
</tr>
<tr>
<td>(Percent of total)</td>
<td>(72.3%)</td>
<td>(78.7%)</td>
</tr>
</tbody>
</table>

Courtesy of Chang-Tai Hsieh, Erik Hurst, Charles I. Jones, and Peter J. Klenow. Used with permission.

$\Rightarrow$ only $\approx 25\%$ of rising female LFP explained by technology
Solving the Model: Output without Frictions

Figure 6: Counterfactuals: Output Growth due to $A, \phi$ versus $\tau$

- Reduced frictions account for 11% – 15% of cumulative output growth
- Output gains smaller when all frictions operate through $\tau^W$, because some wage gaps attributed to taste-based labor mkt. discrimination
- HHJK predict an additional 10% – 14% to be gained from eliminating remaining 2008 frictions
Solving the Model: Relative Wages and Quality

Table 11: Group Changes in Wages

<table>
<thead>
<tr>
<th></th>
<th>Actual Growth</th>
<th>Due to $\tau^h$’s</th>
<th>Due to $\tau^w$’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>White men</td>
<td>77.0 percent</td>
<td>-5.8%</td>
<td>-7.1%</td>
</tr>
<tr>
<td>White women</td>
<td>126.3 percent</td>
<td>41.9%</td>
<td>43.0%</td>
</tr>
<tr>
<td>Black men</td>
<td>143.0 percent</td>
<td>44.6%</td>
<td>44.3%</td>
</tr>
<tr>
<td>Black women</td>
<td>198.1 percent</td>
<td>58.8%</td>
<td>59.5%</td>
</tr>
</tbody>
</table>

White men wages 6% – 7% higher without reduced frictions

Figure 7: Relative Average Quality, White Women vs. White Men

$\tau^h$: women have less human capital; $\tau^w$: women paid below m.p.
Main Takeaway: the Power of Functional Form!

- Basic intuition of HHJK seems very general (ultimately Roy+Mincer!)
  - But model would likely be a disaster without Fréchet
- Authors very upfront that this is not really an empirical paper:
  
  *We freely admit this calculation makes no allowance for model misspecification and thus should be viewed only as an illustration of the potential magnitude of the effect of declining occupational barriers....However, while only illustrative, this calculation captures forces that a simple back-of-the-envelope calculation (based on changing wage gaps alone) does not*

- Very likely a way to test some of the mechanisms (esp. labor market vs. human capital discrimination) with a better identification strategy
  - Real-world Figure 7? Any other ideas?