Macro approaches to consumption smoothing and risk sharing
Using consumption and income data to test models

Robert M. Townsend
MIT
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This lecture focuses on several papers that test various models of consumption smoothing and insurance in the lifecycle (largely from the macro literature). These papers differ in i) the market structure they assume or test (standard incomplete markets (SIM) in which agents can only trade risk free bonds) vs. buffer stock (savings only or limited credit), and on occasion perfect insurance markets, in which all idiosyncratic shocks can be insured, and ii) the data they use.
Summary of the papers covered:
The first paper by Kaplan and Violante assesses the degree of consumption smoothing in the SIM model and finds that it is less than in the data for permanent shocks, i.e., the data for the US exhibits more insurance against permanent shocks. In the SIM model, permanent shocks can only partially be insured against as they will affect agents for the rest of their life. Over the lifecycle, the insurance of permanent shocks increases in the data.
This paper also discusses the Blundell, Pistaferri, and Preston (BPP) methodology. BPP examine the link between income and consumption inequality and are well-known for pioneering a method for constructing a panel data on consumption and income, using an imputation procedure. This procedure involves using variables which are common in the CEX and PSID, to estimate a consumption demand function in the CEX, and then map it to households in the PSID. They then regress consumption on suitable measures of permanent and transitory income shocks to estimate how much of the shocks is passed through to consumption. They find that permanent shocks in the data are partially insured by agents, and transitory shocks are mostly fully insured, except for the very poor. Taxes and transfers play an important insurance role.
Deaton and Paxson take a different look at the data, by examining successive cross-sections of cohorts in the US, the UK, and Taiwan. They first quantify how much consumption and income inequality increases with age within cohorts. They then consider the following models: the PIH, intertemporal allocation with risk aversion, buffer stock model and borrowing constraints, as well as models with risk sharing and insurance between individuals. For each, they derive the implications for the evolution of within-cohort inequality and relate the theory back to the empirical findings.
The paper by Campbell and Deaton documents and attempts to explain the “excess smoothness”, namely the fact that consumption seems to react too little to permanent income shocks, that is there is more insurance in the data than the model would imply. They use disposable income data compiled by Blinder and Deaton (1985) and some data compiled by themselves.
The paper by Krueger and Perri combines Italian Household Survey data (1987 to 2008 - the advantage of which, they claim, is that it contains both income and consumption data), and the PSID to examine the degree of consumption smoothing in response to several income shocks. They also examine the response of wealth to income shocks and introduce heterogeneity between households in terms of asset/business ownership. The response of consumption to income shocks is relatively small, with wealth responding more. This points to either good insurance by the households, with wealth bearing the bulk of the adjustment cost, or to wealth shocks correlated with income shocks.
The common thread among these papers is to examine in the data the degree of consumption smoothing and to adopt a lifecycle perspective, albeit with different market structures. They all face the challenge of incomplete and limited data in developed countries on both consumption and income. The takeaway is that there is some limited consumption smoothing, but that household behavior exhibits several anomalies that cannot be captured by the simple lifecycle model. The weakness of these papers is to not propose a good and robust alternative to the permanent income hypothesis or workhorse lifecycle model.
What is this lecture about?

- How does the macro literature incorporate and test different financial models against each other?
- How to distinguish between market structures and contract arrangements with consumption and income data?
  - example: how to distinguish between permanent income hypothesis with perfect credit markets and a buffer stock model.
- Papers presented here (and two additional ones in the "Appendix" slides) potentially differ in their conclusion but show that empirical work can be productively done, even with limited data.
What are the difficulties?

Difficulty 1: Data limitations make these questions difficult to address

- Need both longitudinal data on income and on a comprehensive measure of consumption.
- Surprisingly perhaps, no such a dataset in the US! Consumption either very aggregated, or income data poor.

Strategies used by authors in the face of this data limitation:

- Constructing synthetic cohorts to merge high quality cross-sectional income and consumption data (Attanasio & Davis 1996).
What are the difficulties?

Difficulty 2: Separate permanent from transitory shocks in finite data

- Need to identify individual income shocks in the data. Given the empirical autocovariance function of individual income, income shocks are combination of very persistent and very transitory shocks (MaCurdy 1982, Abowd & Card 1989, Blundell & Preston 1998).

- In panel data: only total income change observed: how to disentangle shocks of different persistence?

- Strategies used:
  - Use proxies for permanent and transitory income changes (e.g., disability and short unemployment spells, respectively) to separately identify the two shocks (Cochrane 1991, Dynarski & Gruber 1997)
  - Estimate consumption response of households to tax rebates (Souleles 1999, Shapiro & Slemrod 2003), assumed to be perceived as either permanent or transitory change in income by households.
Kaplan and Violante (2010)

Idea

- Ask: do current incomplete-markets macroeconomic frameworks used for quantitative analysis admit the right amount of household insurance? their benchmark is the BPP estimate (see Appendix slides)
- Start with the standard incomplete markets (SIM) model: no access to state-contingent claims, but self-insurance through trading a non-state-contingent bond. (in its life-cycle version)
- Households with CRRA utility, subject to permanent and transitory shocks to earnings while they work, and retirement social security benefits.
- Debt limit: either natural borrowing limit or zero limit.
- Save for life-cycle, and precautionary reasons and their wealth helps in absorbing income shocks.
- Simulate artificial panel from model:
  - (i) How does BPP empirical estimate for consumption smoothing compare to the SIM model one? (equivalently, how much consumption insurance is there in data beyond self-insurance?)
  - (ii) Does BPP methodology actually yield reliable estimates of insurance coefficients? (apply it to their artificial data, for which they know true coefficient)
The SIM model generates insurance coefficient for transitory shocks of 94% in natural borrowing constraints (NBC) economy, and 82% in the zero borrowing constraint (ZBC) economy, which is close to the empirical estimate of 95%. But insurance coefficient for permanent shocks is 22% in the NBC economy, and only 7% in the ZBC economy, much lower than actual 36% one from BPP. Life-cycle pattern of insurance coefficients for permanent shocks is sharply increasing and convex, whereas BPP find no age profile. Hence, model generates too much consumption smoothing for older workers nearing retirement, but too little smoothing for workers in early stages of their life-cycle.
Reliability of estimator proposed by BPP: works very well for transitory shocks, but tends to systematically underestimate true coefficient for permanent shocks which is 23% in both the NBC and ZBC economy. Reason: Their estimation procedure, analogous to IV, exploits orthogonality condition between consumption growth and a particular linear combination of past and future income shocks. Bias of this approximation grows when borrowing constraints are tighter. Empirical insurance coefficients could be even larger.
Explore two extensions to SIM model to generate less sensitivity of consumption to permanent shocks:

1. Allow agents to have some foresight about future income realizations. Advance information does not bridge gap between model and data.

2. Generalize statistical process for earnings: Instead of a random walk (I(1) process), make persistent component an AR(1): For some values of persistence coefficient, SIM model matches data.
Standard Incomplete Markets Model

- No aggregate uncertainty
- Agents work until retirement \( T^{ret} \).
- Probability \( \zeta_t \) of surviving to period \( t \); with \( \zeta_t = 1 \) for all ages before retirement
- Utility:

\[
E_0 \sum_{t=1}^{T} \beta^{t-1} \zeta_t u(C_{it})
\]

- with income process:

\[
\log Y_{it} = \kappa_t + y_{it} \\
y_{it} = z_{it} + \varepsilon_{it} \\
z_{it} = z_{it-1} + \eta_{it}
\]

where \( \kappa \) is a deterministic income profile, common across all households, \( z \) is the permanent shock, \( \varepsilon \) the transitory one. Both \( \varepsilon_{it} \) and \( \eta_{it} \) have zero mean, normally distributed with variances \( \sigma_{\varepsilon} \) and \( \sigma_{\eta} \).
Kaplan and Violante (2010)

Standard Incomplete Markets Model

- Budget constraint:

\[ C_{it} + A_{i,t+1} = (1 + r) A_{it} + Y_{it} \quad \text{if} \quad t < T^{ret} \]

\[ C_{it} + \frac{\zeta_t}{\zeta_{t+1}} A_{i,t+1} = (1 + r) A_{it} + P(Y_i) \quad \text{if} \quad t \geq T^{ret} \]

where: \( A_{it} \) are assets for period \( t \) and \( P \) is the social security payoff function; as a function of lifetime earnings.

- Calibrate model using standard numbers from lifecycle literature (which pieces numbers together from various US data sets such as the PSID, or the CEX).
Deaton and Paxson (1994)

Idea

- PIH implies that consumption of each person follows random walk.
- Implies that dispersion of consumption within a group of people should increase over time (as long as they do not have perfectly correlated income shocks).
- Applies for example to a cohort of people born in the same year: inequality should increase as they age.
- Does NOT mean that aggregate inequality (across all cohorts at a point in time) should increase over time: within cohort result only. (Older people have more dispersion, yet young are constantly replacing the old).
- This increasing inequality can be found in other models than PIH as well.
- Reason for studying this: understand better how economy handles risk (closely related to work by Townsend (1994)).
Deaton and Paxson (1994)

Idea

- Start with evidence: data from the US, Britain and Taiwan, to examine intra-cohort dispersion of consumption, income and earnings over time
- Use 47 annual household surveys across those 3 countries
- Consider then models and contrast their implications for evolution of within-cohort inequality to data. (PIH, intertemporal allocation with risk aversion, buffer stock model with borrowing constraints, models with risk-sharing and insurance between individuals).
Use successive years of cross-sectional household survey data to follow cohorts over time. (not panel data, but for each cohort use representatives from each cross-section).

Example: Taiwanese born in 1945. Means considering those 31 years old in 1976 survey, the 32 years old in the 1977 survey and so on., ending with the 45 years old in 1990 survey.

Actually better than panel data, since no attrition and only summary statistics used, so these are more accurate.

Problem in data: consumption is at household level, earnings and income are individual, and household composition evolves over time. Convert everything into household units.

Fig. 1.—Taiwan: variances of log consumption and age, selected cohorts

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Deaton and Paxson (1994)
Empirical Results - United States

Fig. 2.—United States: variances of log consumption and age, all cohorts

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Fig. 3.—Great Britain: variances of log consumption and age, all cohorts
Fig. 4.—Age effects (and confidence bands) for the variance of log consumption

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Empirical Results - Age Effects on Inequality

- The age profile of inequality for US is close to linear (but rate of dispersion slows down after retirement, around age 60). Regression result: variance of logs within each cohort increases by 0.07 every decade.
- In Taiwan: convex profile, no further widening of inequality after 60. (regression coefficient of 0.08 on decade of age)
- In Britain: slightly convex, also no further widening of inequality after 60. (variance increases by 0.1 for every decade).
- These numbers are extremely large relative to the overall increases in aggregate inequality (across all cohorts).
- Probably demographic changes in those countries, combined with those intra-cohort results can explain large part of the overall increases in inequality.
Deaton and Paxson (1994)
Theoretical Results: Inequality in the PIH

- Intertemporally additive preferences, quadratic per period utility (certainty equivalence)
- Interest rate = discount rate
- Then optimal choice in PIH implies: (see Hall 1978):
  \[ c_{it} = c_{i,t-1} + u_{it} \]
  Assume \( \text{cov}(c_{it-1}, u_{it}) = 0 \)
Hence, the variances over any set of individuals who exist both at $t - 1$ and at $t$ is:

$$\text{var}_t (c) = \text{var}_{t-1} (c) + \sigma_t^2$$

where $\sigma_t^2$ is the period $t$ variance of $u_{it}$. Hence, variance of consumption mechanically increases over time.

In addition, if $c_{i,t-1}$ independent from $u_{it}$ (stronger than zero covariance!), then cross-sectional distribution of consumption at time $t$ is second-order stochastically dominated by the cross-sectional distribution of consumption in any earlier period.

Note importance of 'fixed membership' assumption: variances must be taken over same people at time $t - 1$ and $t$. Is not valid for a society in which old die and are replaced by young ones.
Caveats:

- Random walk model and PIH itself have limitations: for example quadratic utility

- If allow for taste shifters over time or for heterogeneous preferences across individuals, then results could be either strengthened or weakened. (That dispersion in preferences can be due to shifts in household composition for example, which seems important in Taiwan).

- Important caveat is assumption that $cov(c_{it-1}, u_{it}) = 0$ (it is NOT implied by PIH itself for the cross-section, although true for each individual separately).
Inequality increases in the PIH independently from the income process. But income process affects share of the age-inequality profile. Consider a single asset $A_t$:

$$A_t = (1 + r) [A_{t-1} + y_{t-1} - c_{t-1}] \quad (1)$$

where $y$ are earnings, as specified earlier $r$ is real constant rate.

Using terminal condition (assets need to be zero in terminal period $T$):

$$\beta_t c_t = \frac{r}{1 + r} A_t + \frac{r}{1 + r} \sum_{k=0}^{R-t} (1 + r)^{-k} E_t y_{t+k} \quad (2)$$

where

$$\beta_t = 1 - \left[ 1 / (1 + r)^{T-t+1} \right]$$
In the standard infinite horizon PIH, $T = \infty$ and $\beta = 1$.

In finite horizon case: $\beta_t$ is concave and decreasing in $t$.

Using (2) and (1):

$$\beta_t \Delta c_t = \eta_t$$

where: $\eta_t$ is the consumption "innovation":

$$\eta_t = \frac{r}{1 + r} \sum \sum_{k=0}^{R-t} (1 + r)^{-k} (E_t - E_{t-1}) y_{t+k}$$
As a function of the history then:

\[ c_t = c_0 + \sum_{\tau=0}^{t} \beta_{-1}^{\tau} \eta_{\tau} \]

Because innovations are uncorrelated:

\[ \text{var} (c_t) = \text{var} (c_0) + \sum_{\tau=0}^{t} \beta_{-2}^{\tau} \sigma_{\eta_t}^{2} \]

where \( \sigma_{\eta_t}^{2} \) is the variance of age \( t \) consumption innovation \( \eta_t \).
Implications:

1. After retirement, when no more earnings innovations, no more consumption innovations either and growth in consumption inequality should decrease.

   This is what the graphs showed for Britain and Taiwan. US is anomalous and could be due to uncertainty regarding health costs, not covered as well as in the two other countries.
Deaton and Paxson (1994)
Theoretical Results: Shape of the profile

2. Age inequality profile (i.e. cross sectional variation along a cohort) can be either concave or convex up to retirement, depending on persistence of shocks to earnings: concave unless individual earnings are very stationary.

- For example, \( \eta_t = \frac{r \varepsilon_t}{1+r} \) where \( \varepsilon_t \) is white noise. Hence, \( \sigma^2_{\eta_t} \) is constant and \( \text{var} (c_t) = \text{var} (c_0) + \sigma^2_{\eta} \sum_{\tau=0}^{t} \beta^{-2}_{\tau} \) is increasing and convex, since \( \beta_{\tau}^{-2} \) is.
- Persistent example: \( y_t = y_{t-1} + \varepsilon_t - \theta \varepsilon_{t-1} \) with \( 0 \leq \theta \leq 1 \). Then:

\[
\Delta c_t = \beta_t^{-1} \left[ \beta_t^R (1 - \theta) + \theta r (1 + r)^{-1} \right] \varepsilon_t
\]

where \( \beta_t^R = \beta_t = 1 - \left[ 1 / (1+r)^{R-t+1} \right] \) (annuitization factor computed to \( R \)). Inequality-age profile will be concave if \( \beta_t^{-1} \left[ \beta_t^R (1 - \theta) + \theta r (1 + r)^{-1} \right] \) is decreasing in age, which occurs if: \( 1/\theta > r / \left( (1+r) \left[ (1+r)^{T-R} - 1 \right] \right) \) - true unless \( \theta \) very close to 1 (high persistence).
- In data, profiles are not concave. Imply that if the PIH is valid, earnings must contain a large stationary component (or PIH is not valid).
3. PIH also implies that dispersion of income increases with age (up to retirement) and rate of dispersion depends on stochastic income process.

- To see this consider

$$y_t^d = \frac{r}{1 + r} A_t + y_t = \beta_t c_t + s_t$$

where $y_t^d$ is disposable income, as sum of asset income and earnings. Savings $s_t$ is difference between disposable income and annuitized consumption $\beta_t c_t$.

- Savings defined in this way satisfy Campbell's "rainy day" equation (savings is equal to NPV of future expected fall in earnings):

$$s_t = \sum_{k=1}^{R+1-t} (1 + r)^{-k} E_t (-\Delta y_{t+k})$$
Can hence rewrite disposable income as

\[ y_t^d = \beta_t \left( c_0 + \sum_{\tau=1}^{t} \beta_\tau^{-1} \eta_\tau \right) - \sum_{k=1}^{R+1-t} (1 + r)^{-k} E_t \Delta y_{t+k} \]

If stationary earnings: cross-section distribution of earnings will be constant, savings will also be stationary and have a constant cross-section distribution. Disposable income is sum of consumption (integrated process) and savings (stationary) so will also be integrated and its distribution will disperse at the rate of consumption.

This is not borne out in the data, since within-cohort distribution of earnings is dispersing with age.

If nonstationary earnings, earnings are dispersing. Both savings and consumption are integrated, and variance of disposable income (the sum of both) will grow until retirement, but at faster rate than variance of consumption.
Suppose more general, intertemporally separable preferences \( u(c_{it}) \), with \( \delta \) discount rate.

Euler equation drives evolution of consumption:

\[
(1 + r_{t+1}) \lambda (c_{it+1}) = (1 + \delta) \lambda (c_{it}) + u_{it}
\]

where \( \lambda = u' \).

If \( \delta \geq r_{t+1} \), marginal utilities of consumption become more dispersed over time. Can show that if in addition, marginal utility of consumption is concave, variance of consumption increases (sufficient but not necessary condition).

On the other hand, if there is a precautionary motive (\( \lambda \) convex), dispersion of consumption is limited by people’s desire to minimize risk and prudence.
In buffer stock models, liquidity constraints, impatience and stationary earnings can generate an invariant distribution for consumption (constant consumption inequality).

Hence consumption inequality will be less than earnings inequality - this resembles the data for Britain and the US, apart from earnings not being stationary.

With non stationary earnings, buffer stock will be used to smooth out only transitory changes, while long run changes will translate into consumption change. Generates invariant distribution of ratio of consumption, income and assets to earnings, so all four quantities’ variances grow at same rate. This fits Britain and US data.
If no transfers between generations and age distribution in a population is constant, consumption and income inequality remain constant, even though within cohort inequality is increasing. Data confirms that changes in aggregate inequality are much smaller than changes in within-cohort inequality over time.

Relationship between age and inequality implies link between demographic change and distribution of resources. Taiwan for example has experienced relative aging, so now more older people relative to young: and hence, overall inequality has increased.

Results suggest different explanation for the Kuznets curve (that inequality should first increase as development increases, and only then decrease), if economic development is accompanied by a demographic transition from high to low fertility. Eventually this leads to redistribution from young to old, widens inequality, up to point where new stable population distribution is established.
In the data, both macro (at aggregate level) and micro (individual households' level) "excess-smoothness" is observed.

What is excess-smoothness?

Lifecycle model implies that shocks to permanent income fully incorporated in consumption, while innovations to transitory consumption of income not.

- If we can identify the permanent shock to income, it should be translated one to one into consumption.
- Campbell and Deaton (1989): consumption seems to be too smooth - does not react sufficiently to innovations to the permanent component of income.
- Micro evidence in favor of over-smoothing, together with possible explanation in terms of incomplete markets: Attanasio and Pavoni (2009)
Excess sensitivity and excess smoothness are different: Excess sensitivity: how consumption reacts to past (predictable) income shocks. Excess smoothness refers to how consumption reacts to present (unpredictable) income shocks.

Paper documents and tests for Excess smoothness and relates it to "excess-sensitivity".

Core idea: in permanent income model, consumption should not react to anticipated changes in income, but should react to permanent changes in income.

Main Equation: "Saving for a rainy-day Equation", introduced by Campbell

$$s_t = - \sum_{i=0}^{\infty} \frac{1}{(1 + r)^i} E_t (\Delta y_{t+i})$$

that is, savings is the discounted present value of expected future declines in income.

This equation can be given empirical content if specify a process for expectation-formation.
Campbell and Deaton (1989)

- First differences in US labor income well-described by AR(1) with positive autoregressive parameter
  - this means innovations to such a process are "more than permanent" (i.e., forever)

- Changes to permanent income are greater than changes to measured income, so consumption should change by more than measured income
  - Not true in the data: consumption much less variable than income fluctuations.

- Possible explanations they consider:
  - Innovations to labor income are in reality less persistent
  - Consumers have more limited information than econometrician (i.e., expectation formation model is wrong).
Actual equations they work with are in loglinear version: For savings:

\[
\frac{s_t}{y_t} \approx - \sum_{i=1}^{\infty} \rho^i E_t \Delta \log y_{t+i} + \kappa
\]

where \( \kappa \) is a constant. For consumption:

\[
\frac{\Delta c_{t+1}}{y_t} \approx \frac{r}{r - \mu} \sum_{i=1}^{\infty} \rho^i (E_{t+1} - E_t) \Delta \log y_{t+i}
\]

where \( r \) = interest rate and \( \mu \) is the growth rate of real labor income.
Study testable implications for the dynamics of consumption and income of models with moral hazard problem with hidden saving.

Agents typically achieve more insurance than under self insurance with a single asset.

Consumption exhibits 'excess smoothness'

- Equivalent to a violation of the intertemporal budget constraint considered in a Bewley economy (with a single asset).
- Excess smoothness parameter has a structural interpretation in terms of the severity of the moral hazard problem.

Tests of excess smoothness, applied to UK micro data:

- Thanks to theoretical model, can interpret them as tests of the market structure faced by agents.
Appendix Slides

Very useful material to understand the Kaplan and Violante paper better (as well as for its own sake!)
Krueger and Perri (2010): How do households respond to income shocks?

Idea

- Document correlations of consumption and wealth with short run and long run labor income changes
- Find that labor income changes are associated with small consumption changes, and larger wealth changes
- This is less true if the changes are long-term ones
- Implications: labor income shocks have small persistence and can be well insured with simple savings (bond).
How households react to shocks depends on underlying financial structure.
- If no financial markets, consumption bears full weight of adjustment to income shocks
- With complete markets, no effect of transitory shocks on consumption

Important question for macro policies and public policies (social insurance)

Their contribution: don’t only consider consumption adjustment, but also adjustment of wealth in response to income shocks

Hence use the "only" two datasets containing data on consumption, income and wealth (PSID and Italian consumer survey) (by the way - these are NOT the only such datasets!)

Divide households in two groups: households who do own businesses or real estate and households who do not.
Italian survey: For households who do not own wealth nor real estate nondurable consumption changes by about 23 cents in response to a short run (two years) 1 Euro change financial wealth responds by about 17 cents. (Similar in the PSID).

In response to longer run (six years) income changes consumption response becomes stronger, while wealth response becomes weaker.

For households who own real estate or businesses the consumption response to income shocks is much smaller (5 cents) while wealth response is much larger.
Consider two financial models to test which one better accounts for the data: standard PIH model and standard incomplete markets model.

**PIH:** households can freely borrow and save with a risk-free bond; quadratic utility and face both purely transitory and purely permanent shocks.

**Standard incomplete markets model:** CRRA utility (precautionary savings motive), zero borrowing constraint, with same type of shocks.

Show that the co-movement between income, consumption and wealth changes both in the short run and in the long run predicted by the model is consistent with that observed in the data for non-business, non-real estate owners, if transitory shocks are an important source of income changes and if measurement error in income is substantial (which they think is plausible).

Hence, simple PIH model does remarkably well for consumption response.
Krueger and Perri (2010): How do households respond to income shocks?

Models tested

- Long-run response of wealth is informative about model too (see the formal predictions below)

- In models in which the size of precautionary saving motive independent of income realization or wealth level (PIH or with CARA utility and nonbinding borrowing constraints), wealth response should be lower for longer horizon income shocks (keeping magnitude of shock constant)

- In incomplete markets model with CRRA utility (and/or borrowing constraints) wealth response to income shock is increasing with time horizon.

- Again, data more consistent with pure PIH.
Krueger and Perri (2010): How do households respond to income shocks?

Models tested - details of the PIH

- Quadratic per period utility, discount factor $\beta$
- Free lending and borrowing at rate $r$, assume $\beta (1 + r) = 1$
- After tax labor income process:

$$y_t = \bar{y} + z_t + \varepsilon_t + \gamma_t$$
$$z_t = z_{t-1} + \eta_t$$

where $\bar{y}$ is expected household income, $\varepsilon_t \sim N (0, \sigma_{\varepsilon}^2)$ is transitory shock, $\eta_t \sim N (0, \sigma_{\eta}^2)$ is permanent income shock and $\gamma_t \sim N (0, \sigma_{\gamma}^2)$ is measurement error.

- Simple budget constraint (for illustrative purposes):

$$c_t + w_{t+1} = y_t + (1 + r) w_t$$

where $w_t = a_t + e_t$ is total wealth, $c_t$ are expenditures (durable and nondurable, lumped together). Transfers already included in labor income $y_t$. 
Kruweger and Perri (2010): How do households respond to income shocks?

Models tested - details of the PIH

- Responses: (see Deaton 1992) over one period:
  \[
  \Delta c_t = \frac{r}{1 + r} \varepsilon_t + \eta_t \\
  \Delta w_t = \frac{\varepsilon_t}{1 + r} \\
  \Delta y_t = \Delta \varepsilon_t + \eta_t + \Delta \gamma_t
  \]

- Can deduce responses over \( N \) periods:
  \[
  \Delta^N c_t = \sum_{\tau = t-N+1}^{t} \left( \frac{r}{1 + r} \varepsilon_\tau + \eta_\tau \right) \\
  \Delta^N w_t = \sum_{\tau = t-N+1}^{t} \frac{\varepsilon_\tau}{1 + r} \\
  \Delta^N y_t = \sum_{\tau = t-N+1}^{t} \left( \Delta^N \varepsilon_t + \eta_\tau + \Delta^N \gamma_t \right)
  \]
Hence the bivariate regression coefficients are given by:

\[
\beta^N_{c} = \frac{\text{Cov} \left( \Delta^N c_t, \Delta^N y_t \right)}{\text{Var} \left( \Delta^N y_t \right)} = \frac{N\sigma^2_{\eta} + r\sigma^2_{\varepsilon} / (1 + r)}{N\sigma^2_{\eta} + 2 \left( \sigma^2_{\varepsilon} + \sigma^2_{\gamma} \right)} = \frac{NQ + (1 - M) \frac{r}{1+r}}{NQ + 2}
\]

\[
\beta^N_{w} = \frac{\text{Cov} \left( \Delta^N w_t, \Delta^N y_t \right)}{\text{Var} \left( \Delta^N y_t \right)} = \frac{\sigma^2_{\varepsilon} / (1 + r)}{N\sigma^2_{\eta} + 2 \left( \sigma^2_{\varepsilon} + \sigma^2_{\gamma} \right)} = \frac{(1 - M)}{[NQ + 2] (1 + r)}
\]

where \( M = \frac{\sigma^2_{\gamma}}{\sigma^2_{\gamma} + \sigma^2_{\varepsilon}} \) is the share of transitory shock attributed to measurement error, \( Q = \frac{\sigma^2_{\eta}}{\sigma^2_{\gamma} + \sigma^2_{\varepsilon}} \) is ratio of permanent shock to transitory shock.
Larger permanent shock \((Q)\) -> larger consumption response, smaller wealth response.

Increasing \(N\) has exactly same effect as increasing permanent shock ratio \(Q\). (intuition: all transitory shocks are mean-reverting while permanent shocks cumulate. Increasing \(N\) effectively increases persistence of shocks).

Larger measurement error lowers both coefficients (well-known attenuation bias).

Because of certainty equivalence (quadratic utility), size of variances \(\sigma_{\eta}^2\) and \(\sigma_{\epsilon}^2\) per se has no effect.
Utility

\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma} \]

Income process \( y_t = \bar{y}_t \tilde{y}_t \) where \( \tilde{y}_t \) is the stochastic part of income in logs, specified as:

\[
\log(\tilde{y}_t) = z_t + \varepsilon_t \\
z_t = z_{t-1} + \eta_t
\]

with \( \varepsilon_t \sim N\left(-\frac{\sigma^2_\varepsilon}{2}, \sigma^2_\varepsilon\right) \) and \( \eta_t \sim N\left(-\frac{\sigma^2_\eta}{2}, \sigma^2_\eta\right) \) (chosen such that \( E(\tilde{y}_t) = 1 \)).

No analytical solutions exist, unlike PIH, so calibrate it with plausible parameters, solve it and simulate paths. Then run regressions on the data generated from model and compare coefficients to data (both infinite and finite horizon).
Krueger and Perri (2010): How do households respond to income shocks?

Detailed wealth response analysis

- Analyze in detail components of wealth reaction
- All wealth components, but especially real estate and business' assets co-moves very strongly with labor income shocks, for the whole sample.
- Large part of this co-movement may be driven by the 'reverse' correlation between labor income shocks and the prices of real estate (or value of businesses), rather than be a response of wealth accumulation to income shocks.
- Suggest that simple model with only idiosyncratic income shocks, but no shocks to value of assets is not appropriate for households who own real estate or business.
First, let us introduce an important methodology used in BPP. Define the following:

**Income process:** Path of residual log earnings $y_{it}$ (deviations from a deterministic and predictable experience profile common across all households).

$$y_{it} = \sum_{j=0}^{t} a_j' x_{i,t-j}$$

$x$: vector of shocks, $a_j$ is vector of coefficients. Shocks are iid with vector of variances $\sigma$. Formulation incorporates very general processes (like ARIMA).

**Insurance coefficient:** $c_{it}$ is consumption: Insurance coefficient for shock $x_{it}$ is:

$$\phi^x = 1 - \frac{cov(\Delta c_{it}, x_{it})}{var(x_{it})}$$

(both variance and covariance are taken over cross-section of all households). Can also define the insurance coefficient at age $t$, $\phi^x_t$ by taking variance and covariance conditional on all households of age $t$.

**Interpretation:** Share of variance of the shock which does NOT translate into consumption variation.
Methodology

- In model simulated data, straightforward to calculate coefficients. In real data: transitory versus permanent shocks not distinguishable and impossible to estimate from finite panel data.

- Instrumental variables type method: Let $y_i$ be vector of income realizations for household $i$, for all ages and let $g_t^x(y_i)$ be a measurable function of this income history (one for each time and each shock). Identification of insurance coefficient for shock $x$ can be done if can find such a function with

\[
\begin{align*}
\text{var} (x_{it}) &= \text{cov} (\Delta y_{it}, g_t^x(y_i)) \\
\text{cov} (\Delta c_{it}, x_{it}) &= \text{cov} (\Delta c_{it}, g_t^x(y_i))
\end{align*}
\]  

(3)  

(4)  

and then

\[
\phi^x = 1 - \frac{\text{cov} (\Delta c_{it}, g_t^x(y_i))}{\text{cov} (\Delta y_{it}, g_t^x(y_i))}
\]
Verifying (4) requires knowledge of the true data generating process for consumption (i.e., the full model), (3) 'only' requires knowledge of the income generating process.

Note that expression for $1 - \phi^x$ is same as for coefficient from an IV regression of consumption changes on income changes using $g$ as instrument. Condition (4) is hard to check without underlying model because it is like the 'exogeneity assumption' of an instrument.

Need further restrictions/assumptions on the i) income process and ii) model for consumption. BPP is like a special case along those two dimensions.
Income process:

\[ y_{it} = z_{it} + \varepsilon_{it} \]

where \( z_{it} \) follows a unit root process with shock \( \eta_{it} \) (variance \( \sigma_{\eta} \)) and \( \varepsilon_{it} \) is an iid shock (variance \( \sigma_{\varepsilon} \)). Hence:

\[ \Delta y_{it} = \eta_{it} + \Delta \varepsilon_{it} \]

Consumption model:

\[ \text{cov}(\Delta c_{it}, \eta_{i,t+1}) = \text{cov}(\Delta c_{it}, \varepsilon_{i,t+1}) = 0 \]

\[ \text{cov}(\Delta c_{it}, \eta_{i,t-1}) = \text{cov}(\Delta c_{it}, \varepsilon_{i,t-2}) = 0 \]

hence "no foresight" (or no advanced information) and "short memory" (or short history dependance).
For transitory shocks, they set $g_{t}^{\varepsilon} (y_i) = \Delta y_{i,t+1}$ and note that, thanks to previous two assumptions:

$$
\text{cov} (\Delta y_{it}, \Delta y_{i,t+1}) = -\text{var} (\varepsilon_{it})
$$

$$
\text{cov} (\Delta c_{it}, \Delta y_{i,t+1}) = -\text{cov} (\Delta c_{it}, \varepsilon_{it})
$$

and for permanent shock $\eta$, use $g_{t}^{\eta} (y_i) = \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}$ and note that:

$$
\text{cov} (\Delta y_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}) = \text{var} (\eta_{it})
$$

$$
\text{cov} (\Delta c_{it}, \Delta y_{i,t-1} + \Delta y_{it} + \Delta y_{i,t+1}) = \text{cov} (\Delta c_{it}, \eta_{it})
$$

hence those $g$ functions are valid instruments for transitory and permanent shocks respectively.