Problems

1. Maximum gain

For each of the following systems, find the frequency $\omega_m$ for which the magnitude of the gain is greatest.

a. \[ \frac{1}{1 + s + s^2} \]

$\omega_m =$ __________

b. \[ \frac{s}{1 + s + s^2} \]

$\omega_m =$ __________

c. \[ \frac{s^2}{1 + s + s^2} \]

$\omega_m =$ __________

Compare the $\omega_m$ for these systems and make sure that you can explain qualitatively any similarities or differences.
2. Phase

For a second-order system with poles at $-1$ and $-4$ (and no zeros), find the frequency at which the phase is $-90^\circ$, using any method except for the vector method. Then illustrate and confirm that result using the vector method.

\[ \omega = \]
3. CT stability

Consider the following feedback system in which the box represents a causal LTI CT system that is represented by its system function.

\[ X \xrightarrow{+} \frac{K}{s^2 + s - 2} \xrightarrow{-} Y \]

a. Determine the range of \( K \) for which this feedback system is stable.

range of \( K \):

b. Determine the range of \( K \) for which this feedback system has real-valued poles.

range of \( K \):
4. DT stability

Consider the following feedback system in which the box represents a causal LTI DT system that is represented by its system function.

\[ \frac{K}{z^2 + z - 2} \]

\[ X \rightarrow + \rightarrow \frac{K}{z^2 + z - 2} \rightarrow Y \]

a. Determine the range of \( K \) for which this feedback system is stable.

range of \( K \):

b. Determine the range of \( K \) for which this feedback system has real-valued poles.

range of \( K \):
5. Automotive suspension

Wheels are attached to an automobile through a suspension system that is designed to minimize the vibrations of the passenger compartment that result when traveling over bumpy terrain. The suspension system consists of a spring and shock absorber that are both compressed when the wheel passes over a bump, so that the sudden motion of the wheel is not directly transmitted to the passenger compartment. The spring generates a force to hold the passenger compartment at a desired distance above the surface of the road, and the shock absorber adds frictional damping. In this problem, you will determine how much damping is desireable by analyzing a simple model of an automobile’s suspension system shown below.

![Suspension System Diagram]

The model consists of a mass $M$ that represents the mass of the car, which is connected through a spring and dashpot to the wheel. The vertical displacement of the wheel from it’s equilibrium position is taken as the input $x(t)$. The vertical displacement of the mass from it’s equilibrium position is taken as the output $y(t)$. The spring is assumed to obey Hooke’s law, so that the force it generates is a constant $K$ times the amount that the spring is compressed relative to it’s equilibrium compression. The shock absorber is assumed to generate a force that is a constant $B$ times the velocity with which the shock absorber is compressed. Notice that by referring $x(t)$ and $y(t)$ to their equilibrium positions, the force due to gravity can be ignored. Assume that $M = 1$ and $K = 1$.

a. Determine the differential equation that relates the input $x(t)$ and output $y(t)$.

b. Determine and plot the impulse response of the system when $B = 0$. Based on this result, give a physical explanation of the problem that would result if there were no shock absorber in the system.

c. Determine an expression for the smallest positive damping constant $B$ for which the poles of the system have real values. Sketch the impulse response of the system for this value of $B$. Based on this result, give a physical explanation of how the shock absorber improves performance of the suspension system.

d. Consider what would happen if $B$ were very large. Sketch the impulse response for the system if $B = 100$. Describe how this response might be less desireable than that in part c. Provide a physical explanation for how a stiff shock absorber can degrade system performance.
6. Dial tones

Pressing the buttons on a touch-tone phone generates tones that are used for dialing. Each button produces a pair of tones of the form

\[ x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t) \]

where \( f_1 \) and \( f_2 \) code the row and column of the button as shown in the following table.

<table>
<thead>
<tr>
<th>( f_1 ) [Hz]</th>
<th>( f_2 ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>697</td>
<td>1  2  3</td>
</tr>
<tr>
<td>770</td>
<td>4  5  6</td>
</tr>
<tr>
<td>852</td>
<td>7  8  9</td>
</tr>
<tr>
<td>941</td>
<td>*  0  #</td>
</tr>
</tbody>
</table>

This problem concerns the design of a system to detect the row and column numbers that were pressed by analyzing the signal \( x(t) \). The following block diagram illustrates the basic structure of such a system.

\[ x[n] = x(nT) \]

The input \( x(t) \) is first sampled with \( T = 10^{-4} \) seconds. The samples are then passed through LTI systems that generate intermediate signals so that \( y_1[n] \) is large when a button in column 1 is pressed, \( y_2[n] \) is large when a button in column 2 is pressed, and \( y_3[n] \) is large when a button in column 3 is pressed. These intermediate signals are then passed through detectors that determine when the signals are bigger than a threshold value \( \Gamma \). Your task is to design the LTI systems. Each should consist of a system with 2 poles of the form shown in the following pole-zero diagram.

Such systems can be simulated by finding the difference equation that corresponds to the system and then iteratively solving that difference equation.
a. Determine values of $r$ and $\Omega_0$ so that the $h_1[n]$ system generates a large response when the “1” key is pressed and a small response when the “2” or “3” keys are pressed. Your solution should work not only when the input consists of a single key press but also when it consists of sequences of key presses (as when dialing a phone number). Submit hardcopies of your code to generate $y_1[n]$ along with a plot of $y_1[n]$.

b. Describe how the choice of $\Omega_0$ affects the output signal $y_1[n]$.

c. Describe how the choice of $r$ affects the output signal $y_1[n]$. In particular, what limits the maximum acceptable value of $r$? Also, what limits the minimum acceptable value of $r$?