Name: Kerberos
Username:

Please circle your section number:

<table>
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<th>Section</th>
<th>Instructor</th>
<th>Time</th>
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<tr>
<td>1</td>
<td>Marc Baldo</td>
<td>10 am</td>
</tr>
<tr>
<td>2</td>
<td>Marc Baldo</td>
<td>11 am</td>
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<tr>
<td>3</td>
<td>Elfar Adalsteinsson</td>
<td>1 pm</td>
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<tr>
<td>4</td>
<td>Elfar Adalsteinsson</td>
<td>2 pm</td>
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Partial credit will be given for answers that demonstrate some but not all of the important conceptual issues.

Explanations are not required and will not affect your grade.

You have two hours.
Please put your initials on all subsequent sheets.
Enter your answers in the boxes.
This quiz is closed book, but you may use two 8.5 × 11 sheets of paper (four sides total).
No calculators, computers, cell phones, music players, or other aids.

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1. Convolutions  [18 points]

Consider the convolution of two of the following signals.

Determine if each of the following signals can be constructed by convolving \((a \text{ or } b \text{ or } c)\) with \((a \text{ or } b \text{ or } c)\). If it can, indicate which signals should be convolved. If it cannot, put an X in both boxes.

Notice that there are ten possible answers: \((a * a)\), \((a * b)\), \((a * c)\), \((b * a)\), \((b * b)\), \((b * c)\), \((c * a)\), \((c * b)\), \((c * c)\), or \((X,X)\). Notice also that the answer may not be unique.
2. Laplace transforms \[16 \text{ points}\]
Determine if the Laplace transform of each of the following signals exists. If it does, write \text{yes} in the box. If it does not, write \text{no} in the box. If you don’t know, write \text{?} in the box.

Grading: +2 points for each correct answer; −2 points for each incorrect answer; 0 points for each \text{?} or blank response.

\[\begin{align*}
  x_1(t) &= e^{-t}u(t) + e^{-2t}u(t) + e^{-3t}u(t) & X_1(s) \text{ exists? (yes or no or ?): } \\
  x_2(t) &= e^{-t}u(-t) + e^{-2t}u(t) + e^{-3t}u(t) & X_2(s) \text{ exists? (yes or no or ?): } \\
  x_3(t) &= e^{-t}u(t) + e^{-2t}u(-t) + e^{-3t}u(t) & X_3(s) \text{ exists? (yes or no or ?): } \\
  x_4(t) &= e^{-t}u(-t) + e^{-2t}u(-t) + e^{-3t}u(t) & X_4(s) \text{ exists? (yes or no or ?): } \\
  x_5(t) &= e^{-t}u(t) + e^{-2t}u(t) + e^{-3t}u(-t) & X_5(s) \text{ exists? (yes or no or ?): } \\
  x_6(t) &= e^{-t}u(-t) + e^{-2t}u(t) + e^{-3t}u(-t) & X_6(s) \text{ exists? (yes or no or ?): } \\
  x_7(t) &= e^{-t}u(t) + e^{-2t}u(-t) + e^{-3t}u(-t) & X_7(s) \text{ exists? (yes or no or ?): } \\
  x_8(t) &= e^{-t}u(-t) + e^{-2t}u(-t) + e^{-3t}u(-t) & X_8(s) \text{ exists? (yes or no or ?): }
\end{align*}\]
3. **Impulse response** [12 points]

Sketch a block diagram for a CT system with impulse response

\[ h(t) = (1 - te^{-t}) e^{-2t} u(t). \]

The block diagram should contain only adders, gains, and integrators.
4. **Convolutions**  \([20\text{ points}]\)

Sketch the signal that results for each of the following parts.

\[ f_1(t) = \delta(t) + \delta(t-1) + \delta(t-2) \]

\[ g_1(t) = \delta(t) + \delta(t-1) - \delta(t-2) \]

\[ (f_1 \ast g_1)(t) = \]

Label the important features of your results!

\[ \frac{d}{dt} \left( \frac{\partial}{\partial t} \right) \left( f_2(t) \ast g_2(t) \right) \]

\[ \frac{d}{dt} \left( f_2 \ast g_2(t) \right) = \]

Label the important features of your results!
Label the important features of your results!

Given

\[ f_4[n] = 2^n u[-n] \quad \text{and} \quad g_4[n] = \left( \frac{1}{3} \right)^n u[n] \]

enter the following numbers:

\[ (f_4 * g_4)[-2] = \]
\[ (f_4 * g_4)[-1] = \]
\[ (f_4 * g_4)[0] = \]
\[ (f_4 * g_4)[1] = \]
\[ (f_4 * g_4)[2] = \]
5. **Z transform** [16 points]

Let $X(z)$ represent the Z transform of $x[n]$, and let $r_0 < |z| < r_1$ represent its region of convergence (ROC).

Let $x[n]$ be represented as the sum of even and odd parts

$$x[n] = x_e[n] + x_o[n]$$

where $x_e[n] = x_e[-n]$ and $x_o[n] = -x_o[-n]$.

a. Under what conditions does the Z transform of $x_e[n]$ exist?

conditions:
b. Assuming the conditions given in part a, find an expression for the Z transform of $x_e[n]$, including its region of convergence.

\[
\text{Z transform:} \\
\text{ROC:}
\]
6. **DT approximation of a CT system**  

[18 points]

Let $H_{C1}$ represent a causal CT system that is described by

$$\dot{y}_C(t) + 3y_C(t) = x_C(t)$$

where $x_C(t)$ represents the input signal and $y_C(t)$ represents the output signal.

![Diagram](xc(t) \rightarrow H_{C1} \rightarrow y_C(t))

a. Determine the pole(s) of $H_{C1}$, and enter them in the box below.

Your task is to design a causal DT system $H_{D1}$ to approximate the behavior of $H_{C1}$.

![Diagram](xd[n] \rightarrow H_{D1} \rightarrow yD[n])

Let $x_D[n] = x_C(nT)$ and $y_D[n] = y_C(nT)$ where $T$ is a constant that represents the time between samples. Then approximate the derivative as

$$\frac{dy_C(t)}{dt} = \frac{y_C(t + T) - y_C(t)}{T}.$$
b. Determine an expression for the pole(s) of $H_{D1}$, and enter the expression in the box below.


c. Determine the range of values of $T$ for which $H_{D1}$ is stable and enter the range in the box below.


Now consider a second-order causal CT system $H_{C2}$, which is described by
\[ \ddot{y}_C(t) + 100y_C(t) = x_C(t). \]

d. Determine the pole(s) of $H_{C2}$, and enter them in the box below.

Design a causal DT system $H_{D2}$ to approximate the behavior of $H_{C2}$. Approximate derivatives as before:
\[ \dot{y}_C(t) = \frac{dy_C(t)}{dt} = \frac{y_C(t + T) - y_C(t)}{T} \quad \text{and} \quad \frac{d^2 y_C(t)}{dt^2} = \frac{\dot{y}_C(t + T) - \dot{y}_C(t)}{T}. \]
e. Determine an expression for the pole(s) of $H_{D2}$, and enter the expression in the box below.

f. Determine the range of values of $T$ for which $H_{D2}$ stable and enter the range in the box below.