6.004 Computation Structures
Spring 2009

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Semaphores

Problem 1. The following is a set of three interacting processes that can access two shared semaphores:

```c
semaphore U = 3;
semaphore V = 0;
```

<table>
<thead>
<tr>
<th>Process 1</th>
<th>Process 2</th>
<th>Process 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1:wait(U)</td>
<td>L2:wait(V)</td>
<td>L3:wait(V)</td>
</tr>
<tr>
<td>type(&quot;C&quot;)</td>
<td>type(&quot;A&quot;)</td>
<td>type(&quot;D&quot;)</td>
</tr>
<tr>
<td>signal(V)</td>
<td>type(&quot;B&quot;)</td>
<td>goto L3</td>
</tr>
<tr>
<td>goto L1</td>
<td>signal(V)</td>
<td>goto L2</td>
</tr>
</tbody>
</table>

Within each process the statements are executed sequentially, but statements from different processes can be interleaved in any order that's consistent with the constraints imposed by the semaphores. When answering the questions below assume that once execution begins, the processes will be allowed to run until all 3 processes are stuck in a wait() statement, at which point execution is halted.

A. ⭐ Assuming execution is eventually halted, how many C's are printed when the set of processes runs?

Exactly 3. Each time Process 1 executes the "wait(U)" statement, the value of semaphore U is decremented by 1. Since there are no "signal(U)" statements, the loop in Process 1 will execute only 3 times (ie, the initial value of U) and then stall the fourth time "wait(U)" is executed.

B. ⭐ Assuming execution is eventually halted, how many D's are printed when this set of processes runs?

Exactly 3. Process 1 will execute its loop three times (see the answer to the previous question), incrementing "signal(V)" each time through the loop. This will permit "wait(V)" to complete three times. For every "wait(V)" Process 2 executes, it also executes a "signal(V)" so there is no net change in the value of semaphore V caused by Process 2. Process 3 does decrement the value of semaphore V, typing out "D" each time it does so. So Process 3 will eventually loop as many times as Process 1.

C. ⭐ What is the smallest number of A's that might be printed when this set of processes runs?

0. If Process 3 is scheduled immediately after Process 1 executes "signal(V)", then Process 2 might continue being stalled at its "wait(V)" statement and hence never execute its "type" statements.

D. ⭐ Is CABABDDCABCABD a possible output sequence when this set of processes runs?

No. Here are the events implied by the sequence above:
E. ★ Is CABACDBCABDD a possible output sequence when this set of processes runs?

Yes:

F. ★ Is it possible for execution to be halted with either U or V having a non-zero value?

No. If U has a non-zero value, Process 1 will be able to run. If V has a non-zero value, Process 3 will be able to run.

Problem 2. The following pair of processes share a common variable X:

<table>
<thead>
<tr>
<th>Process A</th>
<th>Process B</th>
</tr>
</thead>
<tbody>
<tr>
<td>int Y;</td>
<td>int Z;</td>
</tr>
<tr>
<td>A1: Y = X*2;</td>
<td>B1: Z = X+1;</td>
</tr>
<tr>
<td>A2: X = Y;</td>
<td>B2: X = Z;</td>
</tr>
</tbody>
</table>

X is set to 5 before either process begins execution. As usual, statements within a process are executed sequentially, but statements in process A may execute in any order with respect to statements in process B.
A. How many different values of X are possible after both processes finish executing?

There are four possible values for X. Here are the possible ways in which statements from A and B can be interleaved.

A1 A2 B1 B2: X = 11
A1 B1 A2 B2: X = 6
A1 B1 B2 A2: X = 10
B1 A1 B2 A2: X = 10
B1 A1 A2 B2: X = 6
B1 B2 A1 A2: X = 12

B. Suppose the programs are modified as follows to use a shared binary semaphore S:

Process A                  Process B
int Y;                     int Z;
wait(S);                   wait(S);
A1: Y = X*2;              B1: Z = X+1;
A2: X = Y;                B2: X = Z;
signal(S);                signal(S);

S is set to 1 before either process begins execution and, as before, X is set to 5.

Now, how many different values of X are possible after both processes finish executing?

The semaphore S ensures that, once begun, the statements from either process execute without interrupts. So now the possible ways in which statements from A and B can be interleaved are:

A1 A2 B1 B2: X = 11
B1 B2 A1 A2: X = 12

C. Finally, suppose the programs are modified as follows to use a shared binary semaphore T:

Process A                  Process B
int Y;                     int Z;
A1: Y = X*2;              B1: wait(T);
A2: X = Y;                B2: Z = X+1;
signal(T);                X = Z;

T is set to 0 before either process begins execution and, as before, X is set to 5.

Now, how many different values of X are possible after both processes finish executing?

The semaphore T ensures that all the statements from A finish execution before B begins. So now there is only one way in which statements from A and B can be interleaved:
Problem 3. The following pair of processes share a common set of variables: "counter", "tempA" and "tempB":

<table>
<thead>
<tr>
<th>Process A</th>
<th>Process B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>A1: tempA = counter + 1;</td>
<td>B1: tempB = counter + 2;</td>
</tr>
<tr>
<td>A2: counter = tempA;</td>
<td>B2: counter = tempB;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The variable "counter" initially has the value 10 before either process begins to execute.

A. ⭐ What different values of "counter" are possible when both processes have finished executing? Give an order of execution of statements from processes A and B that would yield each of the values you give. For example, execution order A1, A2, B1, B2 would yield the value 13.

There are three possible values for X. Here are the possible ways in which statements from A and B can be interleaved.

- A1 A2 B1 B2: X = 13
- A1 B1 A2 B2: X = 12
- A1 B1 B2 A2: X = 11
- B1 A1 B2 A2: X = 11
- B1 A1 A2 B2: X = 12
- B1 B2 A1 A2: X = 13

B. ⭐ Modify the above programs for processes A and B by adding appropriate signal and wait operations on the binary semaphore "sync" such that the only possible final value of "counter" is 13. Indicate what should be the initial value of the semaphore "sync".

We need to ensure that A and B run uninterrupted, but it doesn't matter which runs first.

```c
semaphore sync = 1;

Process A
    wait(sync);
    A1: tempA = counter + 1;
    A2: counter = tempA;
    signal(sync);

Process B
    wait(sync);
    B1: tempB = counter + 2;
    B2: counter = tempB;
    signal(sync);
```

C. ⭐ Draw a precedence graph that describes all the possible orderings of executions of statements A1, A2, B1 and B2 that yield the a final value of 11 for "counter".
D. ★ Modify the original programs for processes A and B by adding binary semaphores and signal and wait operations to guarantee that the final result of executing the two processes will be "counter" = 11. Give the initial values for every semaphore you introduce. Try to put the minimum number of constraints on the ordering of statements. In other words, don't just pick one ordering that will yield 11 and enforce that one by means of semaphores; instead, enforce only the essential precedence constraints marked in your solution to question 3.

\[
\text{semaphore } s_1 = 0; \\
\text{semaphore } s_2 = 0; \\
\]

```
Process A
A1: tempA = counter + 1;  \\
    signal(s1);  \\
    wait(s2);  \\
A2: counter = tempA;
```

```
Process B
B1: tempB = counter + 2;  \\
    wait(s1);  \\
    B2: counter = tempB;  \\
    signal(s2);
```

---

Problem 4. The figure below shows two processes that must cooperate in computing N^2 by taking the sum of the first N odd integers.

```
Process P
N = 5;  \\
Sqr = 0;  \\
loopP:  \\
if (N==0)  \\
    Sqr = Sqr + 2*N + 1;  \\
    goto endP;  \\
N = N - 1;  \\
    goto loopP;
endP:  \\
print(Sqr);
```

```
Process Q
loopQ:  \\
if (N==0)  \\
    Sqr = Sqr + 2*N + 1;  \\
    goto loopQ;
```

A. Add appropriate semaphore declarations and signal and wait statements to these programs so that the proper value of Sqr (i.e., 25) will be printed out. Indicate the initial value of every semaphore you add. Insert the semaphore operations so as to preserve the maximum degree of concurrency between the two processes; do not put any nonessential constraints on the ordering of operations. Hint: Two semaphores suffice for a simple and elegant solution.

Looking at the code for process Q, we see that we want it to execute with N = 4, 3, 2, 1, and 0. So we want
the decrement of \( N \) to happen before the first execution of \( Q \). We can use two semaphores to control the production and consumption of "\( N \) values" by the two loops. To achieve maximum concurrency, we'll signal the availability of a new \( N \) value as soon as it's ready and then do as much as possible (i.e., branch back to the beginning of the loop and check the end condition) before waiting for the value to be consumed:

```c
Semaphore P = 1;  // P gets first shot at execution
Semaphore Q = 0;  // Q has to wait for first N value
int N, Sqr;

Process P
N = 5;
Sqr = 0;
loopP:
    if (N==0)
        goto endP;
    wait(P);
    N = N - 1;
    signal(Q);
    goto loopP;
endP:
    wait(P);  // wait for last Q iteration!
    print(Sqr);

Process Q

    wait(Q);
    Sqr = Sqr + 2*N + 1;
    signal(P);
    goto loopQ;
```

Optionally, you could also split \( Sqr = Sqr + 2*N + 1 \) into \( Sqr = Sqr + 1 \) and \( Sqr = Sqr + 2*N \) to slightly improve concurrency.

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**Problem 5.** A computer has three commonly used resources designated A, B and C. Up to three processes designated X, Y and Z run on the computer and each makes periodic use of two of the three resources.

- Process X acquires A, then B, uses both and then releases both.
- Process Y acquires B, then C, uses both and then releases both.
- Process Z acquires C, then A, uses both and then releases both.

**A.** If *two* of these processes are running simultaneously on the machine, can a deadlock occur? If so, describe the deadlock scenario.

No deadlock is possible since one of the two processes will always be able to run to completion.

**B.** Describe a scenario in which deadlock occurs if all *three* processes are running simultaneously on the machine.

All three processes make their first acquisition then hang waiting for a resource that will never become available.
C. Modify the algorithm for acquiring resources so that deadlock cannot occur with three processes running.

If we change Z to acquire A then C, no deadlock can occur.

---

Problem 6. The following is a question about dining computer scientists. There are 6 computer scientists seated at a circular table. There are 3 knives at the table and 3 forks. The knives and forks are placed alternately between the computer scientists. A large bowl of food is placed at the center of the table. The computer scientists are quite hungry, but require both a fork and a knife to eat.

Consider the following policies for eating and indicate for each policy if it can result in deadlock.

- Attempt to grab the fork that sits between you and your neighbor until you are successful.
- Attempt to grab the knife that sits between you and your neighbor until you are successful.
- Eat
- Return the fork.
- Return the knife.

No deadlock is possible since the number of available forks limits the number of knives that can be acquired, i.e., if you have a fork, a knife is guaranteed to be available.

- Attempt to grab any fork on the table until you are successful (if there are many forks grab the closest one).
- Attempt to grab any knife on the table until you are successful (if there are many forks grab the closest one).
- Eat
- Return the knife.
- Return the fork.

No deadlock is possible for the same reason as above.

- Flip a coin to decide if you are going to first try for a fork or a knife.
- Attempt to grab your choice until you are successful (if there are many of that utensil grab the closest one).
- Attempt to grab the other type of utensil until you are successful (if there are many of that utensil grab the closest one).
- Eat
- Return the knife.
- Return the fork.

Deadlock is possible since now it is possible all the philosophers to acquire a utensil and then stall indefinitely waiting for the other utensil to appear.
Problem 7. Gerbitrail is a manufacturer of modular gerbil cages. A Gerbitrail cage is assembled using a catalog of modules, including gerbil "rooms" of various sizes and shapes as well as sections of tubing whose diameter neatly accommodates a single gerbil. A typical cage contains several interconnected rooms, and may house a community of gerbils:

![Diagram of Gerbitrail cage](image)

The Gerbitrail cages are immensely successful, except for one tragic flaw: the dreaded GERBILOCK. Gerbilock is a situation that arises when multiple gerbils enter a tube going in opposite directions, meeting within the tube as shown below:

![Diagram of gerbil lock](image)

Since each gerbil is determined to move forward and there is insufficient room to pass, both gerbils remain gerbilocked forever.

There is, however, hope on the horizon. Gerbitrail has developed little mechanical gates that can be placed at the ends of each tube, and which can both sense and lock out gerbil crossings. All gates are controlled by a single computer. Each gate X has two routines, X_Enter and X_Leave, which are called when a gerbil tries to enter or leave respectively the tube via that gate; the gerbil is not allowed to proceed (i.e., to enter or leave) until the Enter or Leave call returns. Gerbitrail engineers speculate that these routines can solve Gerbilock using semaphores.

They perform the following experiment:
where each of the ggates A and B are controlled by the following code:

```c
semaphore S=???; /* Shared semaphore. */

A_Enter()              /* Handle gerbil entering tube via A */
{ wait(S); }         
A_Leave()              /* Handle gerbil leaving tube via A */
{ signal(S); }        
B_Enter()              /* Handle gerbil entering tube via B */
{ wait(S); }         
B_Leave()              /* Handle gerbil leaving tube via B */
{ signal(S); }
```

A. What is the proper initial value of the semaphore S?

\[ S = 1 \text{ since one gerbil is allowed in the tube.} \]

B. An argument among the Gerbitrail technical staff develops about how the above solution should be extended to more complex cages with multiple tubes. The question under debate is whether separate semaphores should be allocated to each ggate, to each tube, or whether a single semaphore should be shared for all gates. Help resolve the argument. For each proposal, indicate OK if it works, SLOW if it works but becomes burdensome in complex cages, and BAD if it doesn't prevent gerbilock.

Single semaphore shared among all ggates: OK -- SLOW -- BAD

Semaphore for each tube: OK -- SLOW -- BAD

Semaphore for each ggate: OK -- SLOW -- BAD

Single semaphore shared among all ggates: SLOW
Semaphore for each tube: OK
Semaphore for each ggate: BAD

C. Gerbitrail management decides to invest heavily in the revolutionary technology which promises to wipe out the gerbilock threat forever. They hire noted computer expert Nickles Worth to evaluate their ggate approach and suggest improvements. Nickles looks at the simple demonstration above, involving only 2 gerbils and a single tube, and immediately objects. "You are enforcing non-essential constraints on the behavior of these two gerbils", he exclaims.
What non-essential constraint is imposed by the ggate solution involving only 2 gerbils and one tube? Give a specific scenario.

Since only 1 gerbil is allowed in the tube at a time, both gerbils can't go through the tube simultaneously in the same direction even though that would work out okay.

D. Nickles proposes that a new synchronization mechanism, the gerbiphore, be defined to handle the management of Gerbitrail tubes. His proposal involves the implementation of a gerbiphore as a C data structure, and the allocation of a single gerbiphore to each tube in a Gerbitrail cage configuration.

Nickles's proposed implementation is:

```
semaphore mutex=1;

struct gerbiphore { /* definition of "gerbiphore" structure*/
    int dir; /* Direction: 0 means unspecified. */
    int count; /* Number of Gerbils in tube */
} A, B, ...; /* one gerbiphore for each tube */

int Fwd=1, Bkwd=2; /* Direction codes for tube travel */

/* genter(g, dir) called with gerbiphore pointer g and direction dir whenever a gerbil wants to enter the tube attached to g in direction dir */

genter(struct gerbiphore *g, int dir) {
    loop:
        wait(mutex);
        if (g->dir == 0)
            g->dir = dir;  /* If g->dir unassigned, grab it! */
        if (g->dir == dir) {
            g->count = 1 + g->count;  /* One more gerbil in tube. */
            *********;  /* MISSING LINE! */
            return;
        }
        signal(mutex);
        goto loop;
}

/* gleave(g, dir) is called whenever a gerbil leaves the tube attached to gerbiphore g in direction dir. */

gleave(struct gerbiphore *g, int dir) {
    wait(mutex);
    g->count = g->count - 1;
    if (g->count == 0) g->dir = 0;
```
signal(mutex);
}

Unfortunately a blotch of red wine obscures one line of Nickles's code. The team of stain analysts hired to decode the blotch eventually respond with a sizable bill and the report that "it appears to be Ch. Petrus, 1981". Again, your services are needed.

What belongs in place of ********* at the line commented "MISSING LINE"?

signal(mutex). We have to release the mutual exclusion lock before returning from genter.

E. Ben Bitdiddle has been moonlighting at Gerbitrail Research Laboratories, home of the world's largest gerbil population. Ben's duties include computer-related research and shoveling. He is anxious to impress his boss with his research skills, in the hopes that demonstrating such talent will allow him to spend less of his time with a shovel.

Ben observes that the GRL Gerbitrail cage, housing millions of gerbils and involving tens of millions of tubes, is a little sluggish despite its use of Nickles Worth's fancy gerbiphore implementation. Ben studies the problem, and finds that each gerbil spends a surprisingly large amount of time in genter and gleave calls, despite the fact that tubes are infrequently used due to their large number. Ben suspects some inefficiency in the code. Ben focuses on the gleave code, and collects statistics about which line of code in the gleave definition is taking the most CPU time.

Which line of the (4-line) gleave body do you expect to be the most time consuming?

wait(mutex) since that may hang while waiting for other instances of genter or gleave to complete.

F. Identify a class of inessential precedence constraints whose imposition by Nickles's code causes the performance bottleneck observed by Ben.

The mutex semaphore implements a global lock, so you can't manipulate more than one semaphore at a time.

G. Briefly sketch a change to Nickles's code which mitigates the performance bottleneck.

If we use a separate mutex for each gerbiphore than the mutual exclusion each mutex provides will only apply to operations on that gerbiphore.

H. Encouraged by his success (due to your help), Ben decides to try more aggressive performance improvements. He edits the code to gleave, eliminating entirely the calls to wait and signal on the mutex semaphore. He then tries the new code on a 2-gerbil, 1-tube cage.

Will Ben's change work on a 2-gerbil, 1-tube cage? Choose the best answer.

A. It still works fine. Nice work, Ben!
B. Gerbilock may be caused by two nearly simultaneous genter calls.
C. Gerbilock may be caused by two nearly simultaneous gleave calls.
D. Gerbilock may be caused by nearly simultaneous gleave and genter calls, followed by another genter call.
E. Gerbilock can't happen, although the system may fail in other ways.

Problem 8. Similar Software is a software startup whose 24 employees include you and 23 lawyers. Your job is to finish the initial product: the Unicks timesharing system for a single-processor Beta system. Unicks is very similar to the OS code we've seen in lecture, and to a popular workstation OS. To avoid legal entanglements, it incorporates a distinguishing feature: an inter-process communication mechanism call the tube.

A tube provides a flow-controlled communication channel among processes. The system supports at most 100 different tubes, each identified by a unique integer between 0 and 99. The system primitives for communicating via tubes are the Beta SVCs WTube, which writes the nonzero integer in R1 to the tube whose number is in R0, and RTube, which reads a nonzero datum from the tube whose number is passed in R0. Note that tubes can only be used to pass nonzero integers between processes.

The Unicks handlers for WTube and RTube are shown below:

```c
int Tubes[100]; /* max of 100 tubes */

WTube_handler() {
    int TubeNumber = User.R0; /* which tube to write */
    int Datum = User.R1; /* the data to write */

    if (Tubes[TubeNumber] != 0) { /* wait until Tube is empty */
        User.XP = User.XP - 4;
        return;
    } else {
        Tubes[TubeNumber] = Datum; /* tube empty, fill it up! */
    }
}

RTube_handler() {
    int TubeNumber = User.R0; /* which tube to read */

    if (Tubes[TubeNumber] == 0) { /* wait until there's data */
        User.XP = User.XP - 4;
        return;
    } else {
        User.R1 = Tubes[TubeNumber]; /* read the datum */
        Tubes[TubeNumber] = 0; /* mark the tube as empty */
    }
}```
The handlers run as part of the Unicks kernel and will not be interrupted, i.e., handling of interrupts is postponed until the current process returns to user mode. Note that the initial values in the Tubes array are zero, and keep in mind that only nonzero data is to be written to (and read from) each tube.

A. Let Wi be the ith write on a tube, and Rj be the jth read. What precedence constraint(s) does the above implementation enforce between completion of the Wi and Rj?

The code requires that the With write must precede the Rith read, that the Rith read must precede the Wi +1th write. All other precedence relationships can be derived from these.

B. You observe that a process that waits once will waste the remainder of its quantum looping. Suggest a one-line improvement to each of the handlers which will waste less time synchronizing the communication processes.

Each handler could call the scheduler before returning on a read/write fail.

C. Assume, for the remaining questions, that your improvement HAS NOT been implemented; the original code, as shown, in being used.

Since tubes are advertised as a general mechanism for communication among a set of Unix processes, it is important that they work reliably when several processes attempt to read and/or write the simultaneously. S. Quire, the ex-lawyer CEO, has been unable to figure out just what the semantics of tubes are under these circumstances. He finally asks you for help.

Describe what will happen if a process writes an empty tube while multiple processes are waiting to read it. How many processes will read the new value?

Since RTube_handler() is not interruptible, exactly one process will get the new value.

D. Describe what will happen if multiple processes write different values to a tube at about the same time that another process is doing successive reads from that tube. Will each value be read once? Will at least one value be read, but some may be lost? Will garbage (values other than those written) be read?

Since WTube_handler() is not interruptible, no values will be lost. Each value will be read correctly.

E. S. Quire suggests that the interrupt hardware be modified so that timer interrupts (which mark the end of the quantum for the currently running process) are allowed to happen in both user and kernel mode. How would this modification change your answer to parts (C) and (D)?

If the handlers are interruptible, multiple processes may read the same value or garbage (zero). Multiple processes may write over other values before they are read, but will not be read twice if there is only a single reader.
F. Customers have observed that Unicks seems to favor certain processes under some circumstances. In particular, when one process writes data to a tube while processes A, B, and C are waiting to read it, it is typically the case that the amount of data read by A, B, and C will be dramatically different.

Briefly explain the cause for this phenomenon.

If process D writes data and the scheduler calls A, B, C, and D round-robin (in that order), process A has the best chance of having its Rtube call succeed since it runs right after process D has finished calling Wtube.

G. Sketch, in a sentence or two, a plausible strategy for treating the processes more equitably.

If processes have numbers, each tube could remember the last process that read it. When a process writes to the tube again, it could reorder scheduling so that the waiting process with the next higher process number is called next. Another approach would be to use a randomized scheduler, but this creates the possibility that some processes will not be run for a long period of time.

H. Nickles Worth, a consultant to Similar Software, claims that tubes can be used to implement mutual exclusion—that is, to guarantee that at most one process is executing code within a critical section at all times. He offers an example template:

```c
int TubeNumber = 37;    /* tube to be used for mutual exclusion */
WTube(TubeNumber,1);    /* one-time-only initialization */
while () {      /* loop with critical section */
    /* LOCK ACCESS */
    <critical section>
    /* UNLOCK ACCESS */
}
```

where the regions marked LOCK ACCESS and UNLOCK ACCESS use the tube TubeNumber to ensure exclusive access to the code marked <critical section>. These two regions may contain the forms datum=RTube(TubeNumber) and Wtube(TubeNumber,datum) as a C interface to the tube SVCs.

Fill in the LOCK and UNLOCK code above.

LOCK: datum=RTube(TubeNumber)
UNLOCK: Wtube(TubeNumber,datum)