Practical Dielectrics

Reading - Shen and Kong - Ch. 10

Outline

What Holds Liquids Together - electrostatics
How does the Adhesive Tape Work - electrostatics

Electrostatic Breakdown of Dielectrics
Rotary Electrostatic Motor

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Linear Dielectric Slab Actuator

Note: We are going to keep $Q$ constant on the plates

\[ W_s = \frac{Cv^2}{2} = \frac{Q^2}{2C} \]

\[ f = -\frac{\partial W_s}{\partial z} \bigg|_Q \]

\[ f = \frac{Q^2}{2C^2} \frac{dC}{dz} \]

\[ C(\tilde{z}) = \frac{\varepsilon_o \varepsilon_r \tilde{z} W}{s} + \frac{\varepsilon_o (L - \tilde{z}) W}{s} \]
**Force and Differential Pressure**

The differential pressure pushing the dielectric slab into the capacitor is given by:

\[ f = \frac{Q^2 W}{2C^2 \varepsilon} (\varepsilon_r - 1) \varepsilon_0 = \frac{E^2 s W}{2} (\varepsilon_r - 1) \varepsilon_0 \ [N] \]

\[ = (\varepsilon_r - 1) \varepsilon_0 \frac{E^2}{2} W s \]

\[ = \Delta P_e A \]

**differential pressure** pushing the dielectric slab into the capacitor in \([\text{N/m}^2]\) or \([\text{J/m}^3]\) (just the difference of electric field densities on each side)
The 3 Most Common States

“of Earthbound Creatures”

... so what role does electrostatics play in keeping materials together
Induced Dipole Forces between Atoms/Molecules

Electrostatic attraction distorts electron clouds.

Helium atom 1

Helium atom 2

ELECTROSTATIC ATTRACTION DISTORTS ELECTRON CLOUDS

Induced Dipole forces are lower between spherical than between sausage-like molecules (see the boiling points)

Why?

n-Pentane (bp = 309.4 K)
Image in public domain

Neopentane (bp = 282.7 K)
Image in public domain
**What is a Dipole?**

NO external E filed

external E filed stretches the atom/molecule

**Dipole Moment** = \[ \vec{p} = \delta \vec{x} \]

**electric dipole moment,** \( p \) (or **electric dipole** for short), is a measure of the polarity of a system of electric charges. Here \( \vec{x} \) is the displacement vector pointing from the negative charge to the positive charge. This implies that the electric dipole moment vector points from the negative charge to the positive charge. Note that the electric field lines run away from the positive charge and toward the negative charge. There is no inconsistency here, because the electric dipole moment has to do with the positions of the charges, not the field lines.
Dielectric Response of Water

\[ \varepsilon_r \approx 80 \]

More precisely, the relative dielectric constant of H\(_2\)O is:
- 87.9 at 0°C, 78.4 at 25°C, 55.6 at 100°C
- Ice: 99 at -20°C, 171 at -120°C
- Gas: 1.0059 at 100°C, 101.325 kPa

A stream of distilled water
... is attracted to a charged rod (or comb)

Image by Dottie Mae
http://www.flickr.com/photos/dottiemae/5202454566/ on flickr
Crystal structure of ice is hexagonal, resulting in planar, hexagonal snowflakes.

Water in three states: liquid, solid (ice), and (invisible) vapor in air. Clouds are droplets of liquid, condensed from water vapor.

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**Electrostatic Adhesive Surface**

Conducting wires embedded in an insulator produce an electric field in the air just above the surface of the dielectric. If a paper is placed on top of the dielectric, the electrostatic force pulls it closer to the surface.

Typical voltage used to charge the imbedded conductors = 300 Volts
Typical spacing between conductors = 2 mm
Xerography

Image is in the public domain
How Strong is this Force?

The maximum electric field strength is limited by the electrostatic breakdown.

Typically...

\[ E_{max} \approx 10^6 \frac{\text{V}}{\text{m}} \]

\[ \frac{f}{A} = \frac{1}{2} \varepsilon_0 E^2 \approx \]

For small gaps...

\[ E_{max} \approx 10^8 \frac{\text{V}}{\text{m}} \]

\[ \frac{f}{A} = \frac{1}{2} \varepsilon_0 E^2 \approx \]
Dielectric Breakdown

... is similar to ionization of air.

In the insulating region of a capacitor, there is a small number of free electrons due to impurities and other defects. These electrons are accelerated by the electric field and collide with the lattice structure. If the field strength is high enough, the electron generates more electrons with collision, resulting in an avalanche effect or a large current.

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative permittivity</th>
<th>Breakdown E field ($10^6$V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.0</td>
<td>Approximately 3</td>
</tr>
<tr>
<td>Oil</td>
<td>2.3</td>
<td>15</td>
</tr>
<tr>
<td>Paper</td>
<td>1.5-4.0</td>
<td>15</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.7</td>
<td>20</td>
</tr>
<tr>
<td>Glass</td>
<td>6.0</td>
<td>30</td>
</tr>
<tr>
<td>Mica</td>
<td>6.0</td>
<td>200</td>
</tr>
</tbody>
</table>
Electromagnetic Energy Storage

Recall from last week ...

Magnetic

\[
\frac{W_S}{V} = \frac{1}{2} \mu H \cdot H
\]

\[
= \frac{1}{2} B \cdot H
\]

Magnetic machine

Electric

\[
\frac{W_S}{V} = \frac{1}{2} \varepsilon E \cdot E
\]

\[
= \frac{1}{2} D \cdot E
\]

Electric machine
## Electrostatic vs Magnetostatic Actuators

### Magnetic

\[
\frac{f}{A} = \frac{W_S}{V} = \frac{1}{2} \mu H \cdot H
\]

### Electric

\[
\frac{W_S}{V} = \frac{1}{2} \varepsilon E \cdot E
\]

<table>
<thead>
<tr>
<th>Component</th>
<th>Max Field</th>
<th>( \frac{W_S}{V} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic</td>
<td>( H_{max} \approx 1 \text{ T} )</td>
<td>400 kJ/m(^3)</td>
</tr>
<tr>
<td>Electric (Macro)</td>
<td>( E_{max} \approx 10^6 \frac{\text{V}}{\text{m}} )</td>
<td>4.4 J/m(^3)</td>
</tr>
<tr>
<td>Electric (Micro)</td>
<td>( E_{max} \approx 10^8 \frac{\text{V}}{\text{m}} )</td>
<td>44 kJ/m(^3)</td>
</tr>
<tr>
<td>Electric (Bio/Nano)</td>
<td>( E_{max} \approx 10^9 \frac{\text{V}}{\text{m}} )</td>
<td>4.4 MJ/m(^3)</td>
</tr>
<tr>
<td>Gasoline</td>
<td></td>
<td>38 GJ/m(^3)</td>
</tr>
</tbody>
</table>

\[\mu_o = 4\pi \times 10^{-7} \text{ H/m}\]

\[\varepsilon_o = 8.854 \times 10^{-12} \text{ F/m}\]
Rotary Electrostatic Motor (No Dielectric)
(with 4 segments)

- Rotating Metal Plates
- Slip contact

Plate Overlap Area changes
From 0 to \( \frac{1}{2} \pi R^2 \)

If voltage \( v \) is applied across
the plates, torque \( T \) is produced

\[
T = -\frac{\partial W_s}{\partial \theta}
\]

\[
f = \frac{v^2}{2} \frac{dC}{dx}
\]

\[
\tau = \frac{v^2}{2} \frac{dC}{d\theta}
\]

\[
T = \text{power} = \frac{T\omega}{2} =
\]
Rotary Electrostatic Motor (With Dielectric)
(with 4 segments)

- Rotating Dielectric Plates
- No contact to the rotor needed

If voltage $\nu$ is applied across the plates, torque $T$ is produced.

Plate Overlap Area changes
From 0 to $\frac{1}{2} \pi R^2$
Torsional Electrostatic Actuator

\[ E = \frac{v}{rsin\theta} \approx \frac{v}{r\theta} \]

\[ q = \int \epsilon_0 Edr = \int \frac{w + d}{sin \theta} \epsilon_0 \frac{v}{r\theta} ldr = \frac{\epsilon_0 l}{\theta} \ln \left(1 + \frac{wsin\theta}{d}\right) v \]

\[ C(\theta) = \frac{\epsilon_0 l}{\theta} \ln \left(1 + \frac{wsin\theta}{d}\right) \approx \frac{\epsilon_0 lw}{d} \left(1 - \frac{w\theta}{2d}\right), \quad \theta \ll \frac{\pi}{2} \]

\[ \tau = \frac{v^2}{2} \left(-\frac{\epsilon_0 lw}{d}\right) \left(\frac{w}{2d}\right) = -\frac{v^2 \epsilon_0 lw^2}{2d^2} \]
Torsional Electrostatic Actuator (cont.)

\[ f \, dx \quad \leftrightarrow \quad \tau \, d\theta \]

\[ f = \frac{v^2}{2} \frac{dC}{dx} \quad \leftrightarrow \quad \tau = \frac{v^2}{2} \frac{dC}{d\theta} \]

\[ \tau = \frac{v^2}{2} \left( -\frac{\epsilon_o lw}{d} \right) \left( \frac{w}{2d} \right) = -\frac{v^2}{2} \frac{\epsilon_o lw^2}{2d^2} \]

\[ \tau = -\frac{v^2}{2} \frac{\epsilon_o lw^2}{2d^2} \]
Example: Torsional Electrostatic Actuator

\[ E = 10^6 \frac{V}{\text{cm}} \]

\[ \rho = 2300 \frac{\text{g}}{\text{cm}^3} \]

\[ w = 100 \text{ microns} \]

Estimate time required to sweep 24° (+/- 12°)...
KEY TAKEAWAYS

Correspondence between linear and angular coordinate systems

\[ f \, dx \quad \leftrightarrow \quad \tau \, d\theta \]

\[ f = \frac{v^2}{2} \frac{dC}{dx} \quad \leftrightarrow \quad \tau = \frac{v^2}{2} \frac{dC}{d\theta} \]

\[ f = \frac{Q^2}{2C^2} \frac{dC}{dz} \quad \leftrightarrow \quad \tau = \frac{Q^2}{2C^2} \frac{dC}{d\theta} \]

- The maximum electric field strength is limited by the electrostatic breakdown.
- The maximum magnetic field strength is limited by the maximum current allowed through the coils of the electromagnet.
- Since there are no magnetic monopoles, there is no “magnetostatic breakdown.”