Hypothesis testing
Choosing between \( H=H_0 \) and \( H=H_1 \) with minimum \( P(\text{error}) \)

\[
P(H_0 \text{ is true}) = P(H = H_0) = P(H_0) = p_0
\]

\[
P(H_1 \text{ is true}) = P(H = H_1) = P(H_1) = p_1
\]

\( \Rightarrow \) choose more probable hypothesis for min \( P(\text{error}) \)

\[
P(H_1) > P(H_0)
\]

\( \Rightarrow \) choose \( H_1 \) over \( H_0 \)
Again choosing between $H=H_0$ and $H=H_1$ but now given $R=r$, for min $P(\text{error} | R=r)$

\[
P(H_1 | R = r) > P(H_0 | R = r)
\]

Pick whichever hypothesis has maximum a posteriori probability.
Implementing the maximum a posteriori (MAP) rule

\[ P(H_1|R = r) > P(H_0|R = r) \]

\[ P_1 \cdot f_{R|H}(r|H_1) > P_0 \cdot f_{R|H}(r|H_0) \]
Likelihood ratio test (LRT) implementation of MAP rule

\[
\Lambda(r) = \frac{f_{R|H}(r|H_1)}{f_{R|H}(r|H_0)}
\]

\[
\begin{align*}
\text{‘}H_1\text{’} & > p_0 \cdot f_{R|H}(r|H_0) \\
\text{‘}H_0\text{’} & < p_0 \cdot p_1 = \eta
\end{align*}
\]
Binary hypothesis testing (example)
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\[ P_M \quad P_{FA} \quad p_1 P_M + p_0 P_{FA} = P(\text{error}) \]
Terminology

- prevalence \( (p_1) \)
- (conditional) probability of detection, sensitivity, true positive rate, recall
- specificity, true negative rate
- (conditional) probability of false alarm, false positive rate \( (= 1 – \text{specificity}) \)
- (conditional) probability of a miss, false negative rate \( (= 1 – \text{sensitivity}) \)
- positive predictive value, precision
- negative predictive value