Modal solution of undriven CT
LTI state-space models

6.011, Spring 2018
Lec 6
Glucose-insulin system

From Messori et al., IEEE Control Systems Magazine
Feb 2018
UVA/Padova model (FDA approved!)

From Messori et al., IEEE Control Systems Magazine

\[
\begin{align*}
\dot{x}_1(t) &= -k_{gr}x_1(t) + d(t), \\
\dot{x}_2(t) &= k_{gr}x_1(t) - k_{empt}(x_1(t) + x_2(t))x_2(t), \\
\dot{x}_3(t) &= -k_{abs}x_3(t) + k_{empt}(x_1(t) + x_2(t))x_2(t), \\
\dot{x}_4(t) &= EGP(t) + Ra(t) - U_{id}(t) - E(t) - k_1x_4(t) + k_2x_5(t), \\
\dot{x}_5(t) &= -U_{id}(t) + k_1x_4(t) - k_2x_5(t), \\
\dot{x}_6(t) &= -(m_2 + m_4)x_6(t) + m_1x_{10}(t) + k_{a1}x_{11}(t) + k_{a2}x_{12}(t), \\
\dot{x}_7(t) &= -p_{2u}x_7(t) + p_{2u}\left(\frac{x_6(t)}{V_I} - I_b\right), \\
\dot{x}_8(t) &= -k_i x_8(t) + k_i \frac{x_6(t)}{V_I}, \\
\dot{x}_9(t) &= -k_i x_9(t) + k_i x_8(t), \\
\dot{x}_{10}(t) &= -(m_1 + m_3(t))x_{10}(t) + m_2x_6(t), \\
\dot{x}_{11}(t) &= -(k_d + k_{a1})x_{11}(t) + i(t), \\
\dot{x}_{12}(t) &= k_d x_{11}(t) - k_{a2}x_{12}(t), \\
\dot{x}_{13}(t) &= -k_{sc}x_{13}(t) + k_{sc}x_4(t), \\
\dot{x}_{14}(t) &= -n_Gx_{14}(t) + SR_H(t), \\
\dot{x}_{15}(t) &= -k_Hx_{15}(t) + k_H\max\{x_{14}(t) - H_b, 0\}, \\
\dot{x}_{16}(t) &= \dot{SR}_H^s(t),
\end{align*}
\]

From Messori et al., IEEE Control Systems Magazine

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Linearization at an equilibrium yields an LTI model

\[ q(t) = \tilde{q} + \tilde{q}(t), \quad x(t) = \tilde{x} + \tilde{x}(t), \]

\[ \dot{q}(t) = f(q(t), x(t)) \]

\[ \dot{\tilde{q}}(t) \approx \left[ \frac{\partial f}{\partial q} \tilde{q} \right] \tilde{q}(t) + \left[ \frac{\partial f}{\partial x} \tilde{q} \right] \tilde{x}(t) \]

for small perturbations \( \tilde{q}(t) \) and \( \tilde{x}(t) \) from equilibrium.
Phase plane trajectories

State trajectories for different initial conditions

$\begin{align*}
[q_1(t), q_2(t)] &= [-6, 3.05] \\
[q_1(t), q_2(t)] &= [-3.2, 1.5] \\
[q_1(t), q_2(t)] &= [2, -0.9] \\
[q_1(t), q_2(t)] &= [4, -2.1] \\
[q_1(t), q_2(t)] &= [8, -4]
\end{align*}$
Complex eigenvalue pairs (CT case)

If $\lambda_i$ is a (complex) eigenvalue with eigenvector $v_i$, then its complex conjugate $\lambda_i^*$ is also an eigenvalue, with associated eigenvector $v_i^*$.

Write $i = i + j\omega_i$, $v_i = u_i + jv_i$. Then the contribution of the complex pair to the modal solution is

$$\alpha_i v_i e^{it} + \alpha_i^* v_i^* e^{*it} =$$

$$K_i e^{it} \left[ u_i \cos(\omega_it + \theta_i) - w_i \sin(\omega_it + \theta_i) \right]$$
Acoustics and Vibration Animations

Have fun exploring the animations created by

Prof. Dan Russell, Penn State