Lecture 16 - The pn Junction Diode (II)

**Equivalent Circuit Model**

November 3, 2005

Contents:

1. I-V characteristics (*cont.*)
2. Small-signal equivalent circuit model
3. Carrier charge storage: diffusion capacitance

Reading assignment:

Howe and Sodini, Ch. 6, §§6.4, 6.5, 6.9

Announcements:

Quiz 2: 11/16, 7:30-9:30 PM, open book, **must** bring calculator; lectures #10-18.
Key questions

• How does a pn diode look like from a small-signal point of view?

• What are the leading dependences of the small-signal elements?

• In addition to the junction capacitance, are there any other capacitive effects in a pn diode?
1. I-V characteristics (cont.)

Diode current equation:

\[ I = I_o \left( \exp \left( \frac{qV}{kT} \right) - 1 \right) \]

Physics of forward bias:

- potential difference across SCR reduced by \( V \) ⇒ minority carrier injection in QNR’s
- minority carrier diffusion through QNR’s
- minority carrier recombination at surface of QNR’s
- large supply of carriers available for injection

\[ \Rightarrow I \propto e^{qV/kT} \]
Physics of reverse bias:

- potential difference across SCR increased by $V$
  $\Rightarrow$ minority carrier extraction from QNR’s
- minority carrier diffusion through QNR’s
- minority carrier generation at surface of QNR’s
- very small supply of carriers available for extraction
  $\Rightarrow I$ saturates to small value
I-V characteristics: $I = I_o \left( \exp \frac{qV}{kT} - 1 \right)$
Source/drain-body pn diode of NMOSFET:
Key dependences of diode current:

\[ I = qA n_i^2 \left( \frac{1}{N_a W_p - x_p} D_n + \frac{1}{N_d W_n - x_n} \right) \left( \exp \frac{qV}{kT} - 1 \right) \]

- \( I \propto \frac{n_i^2}{N} (\exp \frac{qV}{kT} - 1) \equiv \text{excess minority carrier concentration at edges of SCR} \)

  - in forward bias: \( I \propto \frac{n_i^2}{N} \exp \frac{qV}{kT} \): the more carrier are injected, the more current flows

  - in reverse bias: \( I \propto -\frac{n_i^2}{N} \): the minority carrier concentration drops to negligible values and the current saturates

- \( I \propto D \): faster diffusion \( \Rightarrow \) more current

- \( I \propto \frac{1}{W_{QNR}} \): shorter region to diffuse through \( \Rightarrow \) more current

- \( I \propto A \): bigger diode \( \Rightarrow \) more current
2. Small-signal equivalent circuit model

Examine effect of small signal overlapping bias:

\[ I + i = I_o [\exp \left( \frac{q(V + v)}{kT} \right) - 1] \]

If \( v \) small enough, linearize exponential characteristics:

\[ I + i = I_o (\exp \frac{qV}{kT} \exp \frac{qv}{kT} - 1) \simeq I_o [\exp \frac{qV}{kT} (1 + \frac{qv}{kT}) - 1] \]

\[ = I_o (\exp \frac{qV}{kT} - 1) + I_o (\exp \frac{qV}{kT}) \frac{qv}{kT} \]

Then:

\[ i = \frac{q(I + I_o)}{kT} v \]

From small signal point of view, diode behaves as conductance of value:

\[ g_d = \frac{q(I + I_o)}{kT} \]
Small-signal equivalent circuit model, so far:

\[ g_d \]

\[ g_d \] depends on bias. In forward bias:

\[ g_d \approx \frac{qI}{kT} \]

\( g_d \) is linear in diode current.
Must add capacitance associated with depletion region:

\[
C_j = C_{jo} \sqrt{1 - \frac{V}{\phi_B}}
\]
3. Carrier charge storage: diffusion capacitance

What happens to the majority carriers?

Carrier picture so far:

If in QNR minority carrier concentration ↑ but majority carrier concentration unchanged
⇒ quasi-neutrality is violated.
Quasi-neutrality demands that at every point in QNR:

\[
\text{excess minority carrier concentration} = \text{excess majority carrier concentration}
\]

Mathematically:

\[
p'(x) = p(x) - p_o \simeq n'(x) = n(x) - n_o
\]

Define integrated carrier charge:

\[
q_{Pn} = qA \frac{1}{2} p'(x_n)(W_n - x_n) = qA \frac{W_n - x_n}{2} \frac{n_i^2}{N_d} (\exp \frac{qV}{kT} - 1) = -qN_n
\]
Now examine small increase in $V$:

Small increase in $V \Rightarrow$ small increase in $q_{Pn} \Rightarrow$ small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$C_{dn} = \left. \frac{dq_{Pn}}{dV} \right|_V$$
Can write $q_{Pn}$ in terms of $I_p$ (portion of diode current due to holes in n-QNR):

$$q_{Pn} = \frac{(W_n - x_n)^2}{2D_p} qA n_i^2 \frac{D_p}{N_d W_n - x_n} \left( \exp \frac{qV}{kT} - 1 \right)$$

$$= \frac{(W_n - x_n)^2}{2D_p} I_p$$

Define *transit time* of holes through n-QNR:

$$\tau_{Tp} = \frac{(W_n - x_n)^2}{2D_p}$$

Transit time is *average time for a hole to diffuse through n-QNR* [will discuss in more detail in BJT]

Then:

$$q_{Pn} = \tau_{Tp} I_p$$

and

$$C_{dn} \simeq \frac{q}{kT} \tau_{Tp} I_p$$
Similarly for p-QNR:

\[ q_{Np} = \tau_{Tn}I_n \]

\[ C_{dp} \simeq \frac{q}{kT}\tau_{Tn}I_n \]

where \( \tau_{Tn} \) is \textit{transit time} of electrons through p-QNR:

\[ \tau_{Tn} = \frac{(W_p - x_p)^2}{2D_n} \]

Both capacitors sit in \textit{parallel} \( \Rightarrow \) total diffusion capacitance:

\[ C_d = C_{dn} + C_{dp} = \frac{q}{kT}(\tau_{Tn}I_n + \tau_{Tp}I_p) = \frac{q}{kT}\tau_T I \]

with:

\[ \tau_T = \frac{\tau_{Tn}I_n + \tau_{Tp}I_p}{I} \]

Note that

\[ q_{Pn} + q_{Np} = \tau_{Tn}I_n + \tau_{Tp}I_p = \tau_T I \]
Complete small-signal equivalent circuit model for diode:
Bias dependence of $C_j$ and $C_d$:

- $C_d$ dominates in strong forward bias ($\sim e^{qV/kT}$)
- $C_j$ dominates in reverse bias and small forward bias ($\sim 1/\sqrt{\phi_B - V}$)

  - For strong forward bias, model for $C_j$ invalid (doesn’t blow up)

  - Common ”hack”, let $C_j$ saturate at value corresponding to $V = \frac{\phi_B}{2}$

$$C_{j,max} = \sqrt{2}C_{jo}$$
Key conclusions

Small-signal behavior of diode:

- **conductance**: associated with current-voltage characteristics
  
  \[ g_d \sim I \text{ in forward bias, negligible in reverse bias} \]

- **junction capacitance**: associated with charge modulation in depletion region
  
  \[ C_j \sim \frac{1}{\sqrt{\phi_B - V}} \]

- **diffusion capacitance**: associated with charge storage in QNR’s to keep quasi-neutrality
  
  \[ C_d \sim e^{qV/kT} \]

  \[ C_d \sim I \]