Lecture 12 - Digital Circuits (I)

The inverter

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Reading assignment:

Howe and Sodini, Ch. 5, §§5.1-5.3.2
Key questions

- What are the key figures of merit of logic circuits?
- How can one make a simple inverter using a single MOSFET?
1. Introduction to digital electronics: the inverter

In digital electronics, digitally-encoded information is represented by means of two distinct voltage ranges:

- **logic 0**: \( V_{MIN} \leq V \leq V_{OL} \)
- **logic 1**: \( V_{OH} \leq V \leq V_{MAX} \)
- **undefined logic value**: \( V_{OL} \leq V \leq V_{OH} \).

Logic operations are performed using **logic gates**.

Simplest logic operation of all: *inversion* ⇒ **inverter**
**Ideal inverter:**

\[
\begin{array}{c|c}
\text{IN} & \text{OUT} \\
0 & 1 \\
1 & 0 \\
\end{array}
\]

Circuit representation and ideal transfer function:

Define *switching point* or **logic threshold**:

\[ V_M \equiv \text{input voltage for which } V_{OUT} = V_{IN} \]

- for \( 0 \leq V_{IN} \leq V_M \) \( \Rightarrow \) \( V_{OUT} = V^+ \)

- for \( V_M \leq V_{IN} \leq V^+ \) \( \Rightarrow \) \( V_{OUT} = 0 \)
Key property of ideal inverter: *signal regeneration*

Ideal inverter returns well defined logical outputs (0 or \(V^+\)) even in the presence of considerable noise in \(V_{IN}\) (from voltage spikes, crosstalk, etc.)
"Real" inverter:

In a real inverter, valid logic levels are defined as follows:

- **logic 0:**
  \[
  V_{MIN} \equiv \text{output voltage when } V_{IN} = V^+
  \]
  \[
  V_{OL} \equiv \text{smallest output voltage where slope=-1}
  \]

- **logic 1:**
  \[
  V_{OH} \equiv \text{largest output voltage where slope=-1}
  \]
  \[
  V_{MAX} \equiv \text{output voltage when } V_{IN} = 0
  \]
Two other important voltages:

\[ V_{IL} \equiv \text{smallest input voltage where slope=}-1 \]
\[ V_{IH} \equiv \text{highest input voltage where slope=}-1 \]

To have \textit{signal regeneration}:

range of input values that produce acceptable logic output > range of valid logic values

Key to signal regeneration in inverter: \textit{high voltage gain}
Quantify signal regeneration through *noise margins*.

Consider chain of two inverters:

Define *noise margins*:

\[
NM_H = V_{OH} - V_{IH} \quad \text{noise margin high}
\]

\[
NM_L = V_{IL} - V_{OL} \quad \text{noise margin low}
\]

When signal is within noise margins:

- logic 1 output from first inverter interpreted as logic 1 input by second inverter
- logic 0 output from first inverter interpreted as logic 0 input by second inverter
Simplifications for hand calculations

Hard to compute $A_v = -1$ points in transfer function.

Approximate calculation:

- Assume $V_{OL} \simeq V_{MIN}$ and $V_{OH} \simeq V_{MAX}$
- Trace tangent of transfer function at $V_M$ (slope=small signal voltage gain at $V_M$)
- $V_{IL} \simeq$ intersection of tangent with $V_{OUT} = V_{MAX}$
- $V_{IH} \simeq$ intersection of tangent with $V_{OUT} = V_{MIN}$
- to enhance noise margin: $|A_v(V_M)| \uparrow$
$|A_v(V_M)| \simeq \frac{V_{MAX} - V_M}{V_M - V_{IL}} \Rightarrow V_{IL} \simeq V_M - \frac{V_{MAX} - V_M}{|A_v(V_M)|}$

$|A_v(V_M)| \simeq \frac{V_M - V_{MIN}}{V_{IH} - V_M} \Rightarrow V_{IH} \simeq V_M(1 + \frac{1}{|A_v(V_M)|}) - \frac{V_{MIN}}{|A_v(V_M)|}$

Then:

$N M_L = V_{IL} - V_{OL} \simeq (V_{MAX} - V_{MIN}) - (V_{MAX} - V_M)(1 + \frac{1}{|A_v(V_M)|})$

$N M_H = V_{OH} - V_{IH} \simeq (V_{MAX} - V_{MIN}) - (V_M - V_{MIN})(1 + \frac{1}{|A_v(V_M)|})$

If $|A_v(V_M)| \to \infty$:

$N M_L \to V_M - V_{MIN} \quad N M_H \to V_{MAX} - V_M$
Transient characteristics

Look at inverter switching in the time domain:

\[ t_R \equiv \text{rise time} \text{ between 10\% and 90\% of total swing} \]
\[ t_F \equiv \text{fall time} \text{ between 90\% and 10\% of total swing} \]
\[ t_{PHL} \equiv \text{propagation delay from high-to-low} \text{ between 50\% points} \]
\[ t_{PLH} \equiv \text{propagation delay from low-to-high} \text{ between 50\% points} \]

Propagation delay: \[ t_P = \frac{1}{2}(t_{PHL} + t_{PLH}) \]
Propagation delay: simplification for hand calculations

- Input wavefunction = ideal square wave
- Propagation delay times = delay times to 50% point

- Hand calculations only approximate
- SPICE essential for accurate delay analysis
2. NMOS inverter with resistor pull up

![NMOS inverter diagram]

Features:

- $V_{BS} = 0$ (typically not shown)
- $C_L$ summarizes capacitive loading of following stages (other logic gates, interconnect lines)

Basic operation:

- if $V_{IN} < V_T$, MOSFET OFF $\Rightarrow V_{OUT} = V_{DD}$
- if $V_{IN} > V_T$, MOSFET ON $\Rightarrow V_{OUT}$ small (value set by resistor/nMOS divider)
Transfer function obtained by solving:

\[ I_R = I_D \]

Can solve graphically: I-V characteristics of pull-up resistor on \( I_D \) vs. \( V_{OUT} \) transistor characteristics:
Overlap I-V characteristics of resistor pull-up on I-V characteristics of transistor:

Transfer function:
Logic levels:

$$V_{OUT} = V_{DS}$$

$$V_{MAX} = V_{DD}$$

$$V_{MIN} = 0$$

$$V_{IN} = V_{GS}$$

For $V_{MAX}$, transistor is cut-off, $I_D = 0$:

$$V_{MAX} = V_{DD}$$

For $V_{MIN}$, transistor is in linear regime; solve:

$$I_D = \frac{W}{L} \mu_n C_{ox} \left(V_{DD} - \frac{V_{MIN}}{2} - V_T\right)V_{MIN} = I_R = \frac{V_{DD} - V_{MIN}}{R}$$

For $V_M$, transistor is in saturation; solve:

$$I_D = \frac{W}{2L} \mu_n C_{ox} (V_M - V_T)^2 = I_R = \frac{V_{DD} - V_M}{R}$$

Will continue next lecture with analysis of noise margin and dynamics...
Key conclusions

- Logic circuits must exhibit *noise margins* in which they are immune to noise in input signal.

- Logic circuits must be *regenerative*: able to restore clean logic values even if input is noisy.

- *Propagation delay*: time for logic gate to perform its function.

- Concept of *load line*: graphical technique to visualize transfer characteristics of inverter.

- First-order solution (by hand) of inverter figures of merit easy if regimes of operation of transistor are correctly identified.

- For more accurate solutions, use SPICE (or other circuit CAD tool).