Lecture 23 - Frequency Response of Amplifiers (I)

Common-Source Amplifier

December 1, 2005

Contents:

1. Introduction
2. Intrinsic frequency response of MOSFET
3. Frequency response of common-source amplifier
4. Miller effect

Reading assignment:

Howe and Sodini, Ch. 10, §§10.1-10.4
Key questions

• How does one assess the intrinsic frequency response of a transistor?

• What limits the frequency response of an amplifier?

• What is the "Miller effect"?
1. **Introduction**

Frequency domain is a major consideration in most analog circuits.

Data rates, bandwidths, carrier frequencies all pushing up.

Motivation:

- Processor speeds ↑
- Traffic volume ↑ ⇒ data rates ↑
- More bandwidth available at higher frequencies in the spectrum

![Frequency vs. Bandwidth Diagram](image)

Figure by MIT OCW.
2. Intrinsic frequency response of MOSFET

How does one assess the intrinsic frequency response of a transistor?

\[ f_t \equiv \text{short-circuit current-gain cut-off frequency} \ [GHz] \]

Consider MOSFET biased in saturation regime with small-signal source applied to gate:

\[ v_s \text{ at input } \Rightarrow i_{out}: \text{ transistor effect } \]
\[ \Rightarrow i_{in} \text{ due to gate capacitance} \]

Frequency dependence: \( f \uparrow \Rightarrow i_{in} \uparrow \Rightarrow \left| \frac{i_{out}}{i_{in}} \right| \downarrow \)

\[ f_t \equiv \text{frequency at which } \left| \frac{i_{out}}{i_{in}} \right| = 1 \]
Complete small-signal model in saturation:

\[ i_{in} - v_{gs} j\omega C_{gs} - v_{gs} j\omega C_{gd} = 0 \]

\[ \Rightarrow i_{in} = v_{gs} j\omega (C_{gs} + C_{gd}) \]

node 1:

\[ i_{out} - g_m v_{gs} + v_{gs} j\omega C_{gd} = 0 \]

\[ \Rightarrow i_{out} = v_{gs} (g_m - j\omega C_{gd}) \]
Current gain:

\[
h_{21} = \frac{i_{out}}{i_{in}} = \frac{g_m - j\omega C_{gd}}{j\omega(C_{gs} + C_{gd})}
\]

\[|h_{21}| = \frac{\sqrt{g_m^2 + \omega^2 C_{gd}^2}}{\omega(C_{gs} + C_{gd})}\]

- For low frequency, \(\omega \ll \frac{g_m}{C_{gd}}\),

\[|h_{21}| \approx \frac{g_m}{\omega(C_{gs} + C_{gd})}\]

- For high frequency, \(\omega \gg \frac{g_m}{C_{gd}}\),

\[|h_{21}| \approx \frac{C_{gd}}{C_{gs} + C_{gd}} < 1\]
\[ |h_{21}| \text{ becomes unity at:} \]

\[ \omega_T = 2\pi f_T = \frac{g_m}{C_{gs} + C_{gd}} \]

Then:

\[ f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \]

\[ \star \star \]
Physical interpretation of $f_T$:

Consider:

$$\frac{1}{2\pi f_T} = \frac{C_{gs} + C_{gd}}{g_m} \approx \frac{C_{gs}}{g_m}$$

Plug in device physics expressions for $C_{gs}$ and $g_m$:

$$\frac{1}{2\pi f_T} \approx \frac{C_{gs}}{g_m} = \frac{2}{3}LWC_{ox}W\mu C_{ox}(V_{GS} - V_T) = \frac{L}{\mu \frac{3}{2}V_{GS}-V_T}$$

or

$$\frac{1}{2\pi f_T} \approx \frac{L}{\mu <E_{chan}>} = \frac{L}{<v_{chan}>} = \tau_t$$

$\tau_t \equiv$ transit time from source to drain [s]

Then:

$$f_T \approx \frac{1}{2\pi \tau_t}$$

$f_T$ gives an idea of the intrinsic delay of the transistor: good first-order figure of merit for frequency response.
To reduce $\tau_t$ and increase $f_T$:

- $L \downarrow$: trade-off is cost
- $(V_{GS} - V_T) \uparrow \Rightarrow I_D \uparrow$: trade-off is power
- $\mu \uparrow$: hard to do
- note: $f_T$ independent of $W$ (for constant $V_{GS} - V_T$)

Impact of bias point on $f_T$:

$$
 f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{W}{L} \mu C_{ox} (V_{GS} - V_T) \frac{2\pi}{2\pi(C_{gs} + C_{gd})} = \sqrt{\frac{2W}{L} \mu C_{ox} I_D} \frac{2\pi}{2\pi(C_{gs} + C_{gd})}
$$

In typical MOSFET at typical bias points:

$$
 f_T \sim 5 - 50 \text{ GHz}
$$
$f_t$ of different device technologies

- Fujitsu (02, EDL)
- UIUC (03, EL)
- IBM (03, IPRM)
- IBM (04, VLSI)

- III-V HEMT
- III-V HBT
- SiGe HBT
- Si CMOS
- InGaAs FETs

World record $\sim 560$ GHz
3. Frequency response of common-source amp

Small-signal equivalent circuit model (assuming current source has no parasitic capacitance):

Low-frequency voltage gain:

\[ A_{v,LF} = \frac{v_{out}}{v_s} = -g_m \left( \frac{r_o}{r_{oc} // R_L} \right) = -g_m R'_{out} \]
capacitors hurt bandwidth because:
- current into $C_{gs}, C_{gd}$ makes $v_{gs} < v_s$
- $C_{db}$ diverts current away from $R_{out}'$

node 1: \[ \frac{v_s - v_{gs}}{R_S} - v_{gs} j \omega C_{gs} - (v_{gs} - v_{out}) j \omega C_{gd} = 0 \]

node 2: \[(v_{gs} - v_{out}) j \omega C_{gd} - g_m v_{gs} - v_{out} j \omega C_{db} - \frac{v_{out}}{R_{out}'} = 0 \]

Solve for $v_{gs}$ in 2:

\[ v_{gs} = v_{out} \frac{j \omega (C_{gd} + C_{db}) + \frac{1}{R_{out}'}}{j \omega C_{gd} - g_m} \]

Plug in 1 and solve for $v_{out}/v_s$:

\[ A_v = \frac{-(g_m - j \omega C_{gd}) R_{out}'}{DEN} \]

with

\[ DEN = 1 + j \omega \left\{ R_S C_{gs} + R_S C_{gd} \left[ 1 + R_{out}' \left( \frac{1}{R_S} + g_m \right) \right] + R_{out}' C_{db} \right\} - \omega^2 R_S R_{out}' C_{gs} (C_{gd} + C_{db}) \]

[check that for $\omega = 0$, $A_{v,LF} = -g_m R_{out}'$]
Simplify:

1. Operate at $\omega \ll \omega_T = \frac{g_m}{C_{gs} + C_{gd}} \Rightarrow$

   $$g_m \gg \omega(C_{gs} + C_{gd}) > \omega C_{gs}, \ \omega C_{gd}$$

2. Assume $g_m$ high enough so that

   $$\frac{1}{R_S} + g_m \simeq g_m$$

3. Eliminate $\omega^2$ term in denominator of $A_v$

   $$\Rightarrow \text{worst-case estimation of bandwidth}$$

Then:

$$A_v \simeq \frac{-g_m R'_\text{out}}{1 + j\omega[R_S C_{gs} + R_S C_{gd}(1 + g_m R'_\text{out}) + R'_\text{out} C_{db}]}$$

This has the form:

$$A_v(\omega) = \frac{A_{v,LF}}{1 + j\frac{\omega}{\omega_H}}$$
At $\omega = \omega_H$:

$$|A_v(\omega_H)| = \frac{1}{\sqrt{2}}|A_{v,LF}|$$

$\omega_H$ gives idea of frequency beyond which $|A_v|$ starts rolling off quickly $\Rightarrow$ bandwidth

For common-source amplifier:

$$\omega_H = \frac{1}{R_S C_{gs} + R_S C_{gd}(1 + g_m R'_{out}) + R'_{out} C_{db}}$$

Frequency response of common-source amplifier limited by $C_{gs}$ and $C_{gd}$ shorting out the input, and $C_{db}$ shorting out the output.
Can rewrite as:

\[
f_H = \frac{1}{2\pi \left\{ R_S [C_{gs} + C_{gd}(1 + |A_{v,LF}|)] + R'_{out} C_{db} \right\}}
\]

Compare with:

\[
f_T = \frac{g_m}{2\pi (C_{gs} + C_{gd})}
\]

\[\square \text{ In general: } f_H \ll f_T \text{ due to}
\]

- typically: \( g_m \gg \frac{1}{R_S} \)
- \( C_{db} \) enters \( f_H \) but not \( f_T \)
- presence of \( |A_{v,LF}| \) in denominator

\[\square \text{ To improve bandwidth,}
\]

- \( C_{gs}, C_{gd}, C_{db} \downarrow \Rightarrow \) small transistor with low parasitics
- \( |A_{v,LF}| \downarrow \Rightarrow \) don’t want more gain than really needed

but...

why is it that effect of \( C_{gd} \) on \( f_H \) appears to being amplified by \( 1 + |A_{v,LF}| \)??!!
4. Miller effect

In common-source amplifier, $C_{gd}$ looks much bigger than it really is.

Consider simple voltage-gain stage:

![Diagram](image)

What is the input impedance?

\[ i_{in} = (v_{in} - v_{out})j\omega C \]

But

\[ v_{out} = -A_v v_{in} \]

Then:

\[ i_{in} = v_{in}(1 + A_v)C \]
or

\[ Z_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{j\omega(1 + A_v)C} \]

From input, \( C \), looks much bigger than it really is. This is called the \textit{Miller effect}.

When a capacitor is located across nodes where there is voltage gain, its effect on bandwidth is amplified by the voltage gain \( \Rightarrow \textit{Miller capacitance} \):

\[ C_{Miller} = C(1 + A_v) \]

Why?

\( v_{in} \uparrow \Rightarrow v_{out} = -A_v v_{in} \downarrow \downarrow \Rightarrow (v_{in} - v_{out}) \uparrow \uparrow \Rightarrow i_{in} \uparrow \uparrow \)

In amplifier stages with voltage gain, it is critical to have small capacitance across voltage gain nodes.

As a result of the Miller effect, there is a \textit{fundamental gain-bandwidth tradeoff} in amplifiers.
Key conclusions

• $f_T$ (short-circuit current-gain cut-off frequency): figure of merit to assess intrinsic frequency response of transistors.

• In MOSFET, to first order,

$$f_t = \frac{1}{2\pi \tau_t}$$

where $\tau_t$ is transit time of electrons through channel.

• In common-source amplifier, voltage gain rolls off at high frequency because $C_{gs}$ and $C_{gd}$ short out input and $C_{db}$ shorts out output.

• In common-source amplifier, effect of $C_{gd}$ on bandwidth is magnified by amplifier voltage gain.

• Miller effect: effect of capacitance across voltage gain nodes is magnified by voltage gain
  \[\Rightarrow \text{trade-off between gain and bandwidth.}\]