Announcements
First Hour Exam - Oct. 7, 7:30-9:30 pm; thru 10/2/09, PS #4

Review
Minority carrier flow in QNRs: 1. $L_{\text{min}} << w$, 2. $L_{\text{min}} >> w$

I-V relationship for an abrupt p-n junction
Assume: 1. Low level injection
2. All applied voltage appears across junction:
3. Majority carriers in quasi-equilibrium with barrier
4. Negligible SCL generation and recombination

Relate minority populations at QNR edges, -$x_p$ and $x_n$, to $v_{AB}$
Use $n'(-x_p)$, $p'(x_n)$ to find hole and electron currents in QNRs
Connect currents across SCL to get total junction current, $i_D$

Features and limitations of the model
Engineering the minority carrier injection across a junction
Deviations at low and high current levels
Deviations at large reverse bias
**QNR Flow:** Uniform doping, non-uniform LL injection

We use the 5 QNR flow conditions* to simplify our 5 equations... (assuming a p-type sample)

\[
\frac{\partial p(x,t)}{\partial t} + \frac{1}{q} \frac{\partial J_h(x,t)}{\partial x} = \frac{\partial n(x,t)}{\partial t} - \frac{1}{q} \frac{\partial J_e(x,t)}{\partial x} \approx g_L(x,t) - \left[ n(x,t)p(x,t) - n_i^2 \right] r(t)
\]

\[
J_h(x,t) = q \mu_h p(x,t) E(x,t) - q D_h \frac{\partial p(x,t)}{\partial x} \approx q \mu_h p_o(x) E(x,t) - q D_h \frac{dn'(x,t)}{dx}
\]

\[
J_e(x,t) = q \mu_e n(x,t) E(x,t) + q D_e \frac{\partial n(x,t)}{\partial x} \Rightarrow J_e(x,t) \approx q D_e \frac{dn'(x,t)}{dx}
\]

\[
\frac{q}{\varepsilon(x)} \left[ p(x,t) - n(x,t) + N_d^+(x) - N_a^-(x) \right] = \frac{d E_x(x,t)}{dx} \approx \frac{q}{\varepsilon} \left[ p'(x,t) - n'(x,t) \right]
\]

..and end up with one equation in \( n' \): the static diffusion equation!

\[
\frac{d^2 n'(x,t)}{dx^2} - \frac{n'(x,t)}{D_e \tau_e} = -\frac{1}{D_e} g_L(x,t)
\]

* Five assumptions that define flow problems AND should be validated at the end.
**QNR Flow, cont.:** Solving the steady state diffusion equation

The steady state diffusion equation in **p-type** material is:

\[
\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = -\frac{1}{D_e} g_L(x)
\]

and for **n-type** material it is:

\[
\frac{d^2 p'(x)}{dx^2} - \frac{p'(x)}{L_h^2} = -\frac{1}{D_h} g_L(x)
\]

In writing these expressions we have introduced \(L_e\) and \(L_h\), the minority carrier diffusion lengths for holes and electrons, as:

\[
L_e \equiv \sqrt{D_e \tau_e} \quad \quad \quad L_h \equiv \sqrt{D_h \tau_h}
\]

We'll see that the minority carrier diffusion length tells us how far the average minority carrier diffuses before it recombines.

In a basic p-n diode, we have \(g_L = 0\) which means we only need the homogenous solutions, i.e. expressions that satisfy:

\[
\text{n-side:} \quad \frac{d^2 p'(x)}{dx^2} - \frac{p'(x)}{L_h^2} = 0 \quad \quad \quad \text{p-side:} \quad \frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = 0
\]
QNR Flow, cont.: Solving the steady state diffusion equation

We seldom care about this general result. Instead, we find that most diodes fall into one of two cases:

Case I - Long-base diode: $w_n \gg L_h$
Case II - Short-base diode: $L_h \gg w_n$

Case I: When $w_n \gg L_h$, which is the situation in an LED, for example, the solution is

$$p'(x) \approx p'(x_n) e^{-(x-x_n)/L_h} \quad \text{for} \quad x_n \leq x \leq w_n$$

This profile decays from $p'(x_n)$ to 0 exponentially as $e^{-x/L_h}$.

The corresponding hole current for $x_n \leq x \leq w_n$ in Case I is

$$J_h(x) \approx -qD_h \frac{dp'(x)}{dx} = \frac{qD_h}{L_h} p'(x_n) e^{-(x-x_n)/L_h} \quad \text{for} \quad x_n \leq x \leq w_n$$

The current decays to zero also, indicating that all of the excess minority carriers have recombined before getting to the contact.
QNR Flow, cont.: Solving the steady state diffusion equation

**Case II:** When \( L_h \gg w_n \), which is the situation in integrated Si diodes, for example, the differential equation simplifies to:

\[
\frac{d^2 p'(x)}{dx^2} = \frac{p'(x)}{L_h^2} \approx 0
\]

We see immediately that \( p'(x) \) is linear:

\[
p'(x) = A x + B
\]

Fitting the boundary conditions we find:

\[
p'(x) \approx p'(x_n) \left[ 1 - \left( \frac{x - x_n}{w_n - x_n} \right) \right]
\]

for \( x_n \leq x \leq w_n \)

This profile is a straight line, decreasing from \( p'(x_n) \) at \( x_n \) to 0 at \( w_n \).

In Case II the current is constant for \( x_n \leq x \leq w_n \):

\[
J_h(x) \approx -qD_h \frac{dp'(x)}{dx} = \frac{qD_h}{w_n - x_n} p'(x_n)
\]

for \( x_n \leq x \leq w_n \)

The constant current indicates that no carriers recombine before reaching the contact.
QNR Flow, cont.: Uniform doping, non-uniform LL injection

Sketching and comparing the limiting cases: $w_n >> L_h$, $w_n << L_h$

Case I - Long base: $w_n >> L_n$ (the situation in LEDs)

Case II - Short base: $w_n << L_n$ (the situation in most Si diodes and transistors)
QNR Flow, cont.: Uniform doping, non-uniform LL injection

The four other unknowns

⇒ In n-type the steady state diffusion equation gives \( p' \).
⇒ Knowing \( p' \), we can easily get \( n' \), \( J_e \), \( J_h \), and \( E_x \):

First find \( J_h \):\[
J_h(x) \approx -qD_h \frac{dp'(x)}{dx}
\]

Then find \( J_e \):\[
J_e(x) = J_{Tot} - J_h(x)
\]

Next find \( E_x \):\[
E_x(x) \approx \frac{1}{q\mu_e n_o} \left[ J_e(x) + \frac{D_e}{D_h} J_h(x) \right]
\]

Then find \( n' \):\[
n'(x) \approx p'(x) - \frac{\varepsilon}{q} \frac{dE_x(x)}{dx}
\]

Finally, go back and check that all of the five conditions are met by the solution.

✓ Once we solve the diffusion equation and get the minority carrier excess we know everything.
**Current flow:** finding the relationship between $i_D$ and $v_{AB}$

**There are two pieces to the problem:**

- Minority carrier flow in the QNRs is what limits the current.
- Carrier equilibrium across the SCR determines $n'(\text{-}x_p)$ and $p'(x_n)$, the boundary conditions of the QNR minority carrier flow problems.

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**Diagram Description:**

- **Uniform p-type**
- **_uniform n-type**
- **Ohmic contact**
- **Quasineutral region I**
- **Space charge region**
- **Quasineutral region II**

**Minority carrier flow here determines the electron current**

**The values of $n'$ at $\text{-}x_p$ and $p'$ at $x_n$ are established here.**

**Minority carrier flow here determines the hole current**
The p-n Junction Diode: the game plan for getting $i_D(v_{AB})$

We have two QNR's and a flow problem in each:

If we knew $n'(-x_p)$ and $p'(x_n)$, we could solve the flow problems and we could get $n'(x)$ for $-w_p<x<-x_p$, and $p'(x)$ for $x_n<x<w_n$ ...
...and knowing $n'(x)$ for $-w_p < x < x_p$, and $p'(x)$ for $x_n < x < w_n$, we can find $J_e(x)$ for $-w_p < x < x_p$, and $J_h(x)$ for $x_n < x < w_n$.

Having $J_e(x)$ for $-w_p < x < x_p$, and $J_h(x)$ for $x_n < x < w_n$, we can get $i_D$ because we will argue that $i_D(v_{AB}) = A[J_e(-x_p, v_{AB}) + J_h(x_n, v_{AB})]$

...but first we need to know $n'(-x_p, v_{AB})$ and $p'(x_n, v_{AB})$.

We will do this now.
The impact of the barrier height change on the carrier populations and fluxes:

**Unbiased junction**
- Population in equilibrium with barrier

**Forward bias on junction**
- Barrier lowered so carriers to left can cross over it.
- The flux is limited by how fast they diffuse in the QNR.

**Reverse bias on junction**
- Barrier raised so the few carriers on top spill back down.
Majority carriers against the junction barrier

Zero applied bias, $v_{AB} = 0$; thermal equilibrium barrier

Hole potential energy, $q\phi$

Notice that:

$$p_o(x \geq x_n) = n_i e^{-q\phi_n/kT} = n_i^2/N_{Dn}$$

$$p_o(x \leq -x_p) = n_i e^{-q\phi_p/kT} = N_{Ap}$$

$$p_o(x_p < x < x_n) = n_i e^{-q\phi(x)/kT}$$

"The holes are in equilibrium with the barrier."
Boundary conditions at the edges of the space charge layer:
What are $n'(-x_p)$ and $p'(x_n)$?

Begin by looking at the situation in thermal equilibrium, where we have:

$$p_o(-x_p) = N_{Ap} \quad \text{and} \quad p_o(x_n) = \frac{n_i^2}{N_{Dn}}$$

If the population of holes at the top of the potential “hill” is related to the population at the bottom by a Boltzmann factor, then we should also find that:

$$p_o(x_n) = p_o(-x_p) e^{-q \phi_b / kT}$$

Do we?

$$\phi_b = \frac{kT}{q} \ln \frac{N_{Ap} N_{Dn}}{n_i^2} \Rightarrow \frac{n_i^2}{N_{Dn}} = N_{Ap} e^{-q \phi_b / kT}$$

Thus:

$$p_o(x_n) = \frac{n_i^2}{N_{Dn}} = N_{Ap} e^{-q \phi_b / kT} = p_o(-x_p) e^{-q \phi_b / kT}$$

**YES, we do, and the Boltzmann relationship holds.**
Majority carriers against the junction barrier

Forward bias, $v_{AB} > 0$; barrier lowered, carriers spill over

We say the holes are still in equilibrium with the barrier at $x_n$:

$$p(x_n) = N_{Ap} e^{-q(\phi_B - v_{AB})/kT} = \frac{n_i^2}{N_{Dn}} e^{+q v_{AB} / kT}$$
Majority carriers against the junction barrier

Reverse bias, $v_{AB} < 0$; barrier raised, carriers spill back

$p(x < -x_p) \approx N_{Ap}$

Again the holes maintain equilibrium with the barrier until $x_n$:

$p(x_n) = N_{Ap} e^{-q(\phi_B + v_{AB})/kT} = \frac{n_i^2}{N_{Dn}} e^{+qv_{AB}/kT}$

And we have the same expression for $p(x_n)$. 
What are \( n'(-x_p) \) and \( p'(x_n) \) with \( v_{AB} \) applied?

We propose that the majority carrier populations on either side are still related by the Boltzmann factor,* which is now:
\[
\exp[-q(\phi_b-v_{AB})/kT]
\]
Thus:
\[
p(x_n) = p(-x_p)e^{-q[\phi_b-v_{AB}]/kT}
\]
Under low level injection conditions, the majority carrier population is unchanged, so \( p(-x_p) \) remains \( N_{Ap} \), so:
\[
p(x_n) = N_{Ap}e^{-q[\phi_b-v_{AB}]/kT} = \frac{n_i^2}{N_{Dn}}e^{qv_{AB}/kT}
\]
And the excess population we seek is:
\[
p'(x_n) = p(x_n) - p_{on} = \frac{n_i^2}{N_{Dn}}e^{qv_{AB}/kT} - 1
\]
Similarly at \(-x_p\):
\[
n'(-x_p) = \frac{n_i^2}{N_{Ap}}e^{qv_{AB}/kT} - 1
\]
* We are assuming that the majority carriers can get across the SCL much faster than they can diffuse away as minority carriers, i.e., that diffusion is the bottleneck!
**Biased p-n junctions:** current flow, cont.

What is the current, \( i_D \)?

Knowing \( p'(x_n) \) and \( n'(-x_p) \), we know:

\[
J_h(x) \quad \text{for} \quad x_n < x < w_n
\]
and

\[
J_e(x) \quad \text{for} \quad -w_p < x < -x_p
\]

But we still don’t know the total current because we don’t know both currents at the same position, \( x \):

\[
i_D = A J_{TOT} = A \left[ J_h(x) + J_e(x) \right]
\]

Have to be at same “\( x \)”

To proceed we make the assumption that there is negligible recombination of holes and electrons in the depletion region, so what goes in comes out and:

\[
J_h(x_n) = J_h(-x_p) \quad \text{and} \quad J_e(x_n) = J_e(-x_p)
\]

With this assumption, we can write:

\[
i_D = A J_{TOT} = A \left[ J_h(x_n) + J_e(-x_p) \right]
\]

Values at edges of SCL
**Biased p-n junctions:** current flow, cont.

What is the current, \( i_D \), cont.?

Both \( J_n(x_n) \) and \( J_e(-x_p) \), are proportional to \( p'(x_n) \) and \( n'(-x_p) \), respectively, which in turn are both proportional to \( (e^{qv/kT} - 1) \):

\[
J_h(x_n) \propto p'(x_n) \propto e^{qv_{AB}/kT} - 1 \quad \text{and} \quad J_e(-x_p) \propto n'(x_p) \propto e^{qv_{AB}/kT} - 1
\]

Thus the diode current is also proportional to \( (e^{qv/kT} - 1) \):

\[
i_D = A \left[ J_h(x_n) + J_e(-x_p) \right] \propto e^{qv_{AB}/kT} - 1 \quad \Rightarrow \quad i_D = I_s \left[ e^{qv_{AB}/kT} - 1 \right]
\]

(\( I_s \) is called the reverse saturation current of the diode.)

**Notice:** The non-linearity, i.e., the exponential dependence of the diode current on voltage, arises because of the exponential dependence of the minority carrier populations the edges of the space charge layer (depletion region). The flow problems themselves are linear.
Biased p-n junctions: current flow, cont.

The saturation current of three diode types: $I_S$'s dependence on the relative sizes of $w$ and $L_{\text{min}}$

**Short-base diode, $w_n << L_h$, $w_p << L_e$:**

$$J_h(x_n) = q \frac{n_i^2}{N_{Dn}} \frac{D_h}{(w_n - x_n)} \left[e^{qV_{AB}/kT} - 1\right]$$

$$J_e(-x_p) = q \frac{n_i^2}{N_{Ap}} \frac{D_e}{(w_p - x_p)} \left[e^{qV_{AB}/kT} - 1\right]$$

$$i_D = Aq n_i^2 \left[\frac{D_h}{N_{Dn}(w_n - x_n)} + \frac{D_e}{N_{Ap}(w_p - x_p)}\right] \left[\frac{e^{qV_{AB}/kT}}{e^{qV_{AB}/kT} - 1}\right]$$

**Long-base diode, $w_n >> L_h$, $w_p >> L_e$:**

$$J_h(x_n) = q \frac{n_i^2}{N_{Dn}} \frac{D_h}{L_h} \left[e^{qV_{AB}/kT} - 1\right]$$

$$J_e(-x_p) = q \frac{n_i^2}{N_{Ap}} \frac{D_e}{L_e} \left[e^{qV_{AB}/kT} - 1\right]$$

$$i_D = Aq n_i^2 \left[\frac{D_h}{N_{Dn}L_h} + \frac{D_e}{N_{Ap}L_e}\right] \left[\frac{e^{qV_{AB}/kT}}{e^{qV_{AB}/kT} - 1}\right]$$

**General diode:**

$$i_D = Aq n_i^2 \left[\frac{D_h}{N_{Dn}w_{n,\text{eff}}} + \frac{D_e}{N_{Ap}w_{p,\text{eff}}}\right] \left[\frac{e^{qV_{AB}/kT}}{e^{qV_{AB}/kT} - 1}\right]$$

- **Hole injection into n-side**
- **Electron injection into p-side**

Note: $w_{n,\text{eff}} = L_h \tanh(w_n - x_n)$, $w_{p,\text{eff}} = L_e \tanh(w_p - x_p)$
**Biased p-n junctions:** current flow, cont.

The ideal exponential diode

- **General expression:**
  \[ i_D = I_S (e^{qV_{AB}/kT} - 1) \]

- **Forward bias, \(|V_{AB}| > kT/q|:**
  \[ i_D \approx I_S e^{qV_{AB}/kT} \]
  Current increases 10x for every 60 mV increase in \(V_{AB}\).

- **Reverse bias, \(|V_{AB}| > kT/q|:**
  Current saturates at \(I_S\).
  \[ i_D = -I_S \]

Ref: Adapted from Figure 18 in S. M. Sze, “Physics of Semiconductor Devices” 1st. Ed (Wiley, 1969)
**Biased p-n junctions: current flow, cont.**

**Limitations of the model**

NOTE: This figure is a bit exaggerated, but it makes the point.

- **Large forward bias:**
  - Sub-exponential increase
  - High level injection (c)
  - Series voltage drop (d)

- **Large reverse bias:**
  - Abrupt, rapid increase
  - Non-destructive breakdown

- **Very low bias levels:**
  - Excess current seen
  - SCL generation and recombination (a, e)

Ref: Figure 18 in S. M. Sze, “Physics of Semiconductor Devices” 1st Ed (Wiley, 1969)
Asymmetrically doped junctions: an important special case

Depletion region impacts/issues

A p+-n junction ($N_{Ap} \gg N_{Dn}$):

$$x_n \gg x_p , \quad w \approx x_n = \sqrt{\frac{2\varepsilon_{Si} (\phi_b - v_{AB})}{qN_Dn}} , \quad |E_{pk}| \approx \sqrt{\frac{2q (\phi_b - v_{AB}) N_{Dn}}{\varepsilon_{Si}}}$$

An n+-p junction ($N_{Dn} \gg N_{Ap}$):

$$x_p \gg x_n , \quad w \approx x_p = \sqrt{\frac{2\varepsilon_{Si} (\phi_b - v_{AB})}{qN_{Ap}}} , \quad |E_{pk}| \approx \sqrt{\frac{2q (\phi_b - v_{AB}) N_{Ap}}{\varepsilon_{Si}}}$$

Note that in both cases the depletion region is predominately on the lightly doped side, and it is the doping level of the more lightly doped junction that matters (i.e., dominates).

Note also that as the doping level increases the depletion width decreases and the peak E-field increases. [This is also true in symmetrical diodes.]

Two very important and useful observations!!
Asymmetrically doped junctions: an important special case

Current flow impact/issues

A p+-n junction \((N_{Ap} > N_{Dn})\):

\[
i_D = Aqn_i^2 \left[ \frac{D_h}{N_{Dn} w_{n,eff}} + \frac{D_e}{N_{Ap} w_{p,eff}} \right] \left[ e^{qV_{AB}/kT} - 1 \right] \approx Aqn_i^2 \frac{D_h}{N_{Dn} w_{n,eff}} \left[ e^{qV_{AB}/kT} - 1 \right]
\]

Hole injection into n-side

An n+-p junction \((N_{Dn} > N_{Ap})\):

\[
i_D = Aqn_i^2 \left[ \frac{D_h}{N_{Dn} w_{n,eff}} + \frac{D_e}{N_{Ap} w_{p,eff}} \right] \left[ e^{qV_{AB}/kT} - 1 \right] \approx Aqn_i^2 \frac{D_e}{N_{Ap} w_{p,eff}} \left[ e^{qV_{AB}/kT} - 1 \right]
\]

Electron injection into p-side

Note that in both cases the minority carrier injection is predominately into the lightly doped side.

Note also that it is the doping level of the more lightly doped junction that determines the magnitude of the current, and as the doping level on the lightly doped side decreases, the magnitude of the current increases.

Two very important and useful observations!!
**6.012 - Microelectronic Devices and Circuits**

**Lecture 6 - p-n Junctions: I-V Relationship**

- **I-V relationship for an abrupt p-n junction**
  Focus is on minority carrier diffusion on either side of SCL
  Voltage across SCL sets excess populations $-x_p$ and $x_n$:
  \[
  n'(x_p) = n_{po} e^{-q(f_b - v_{AB})/kT} - n_{po} = n_{po}(e^{qv_{AB}/kT} - 1) = (n_i^2/N_{Ap})(e^{qv_{AB}/kT} - 1)
  \]
  \[
  p'(x_n) = p_{no} e^{-q(f_b - v_{AB})/kT} - p_{no} = p_{no}(e^{qv_{AB}/kT} - 1) = (n_i^2/N_{Dn})(e^{qv_{AB}/kT} - 1)
  \]

- **Flow problems in QNR regions give minority currents:**
  \[
  J_e(-w_p < x < -x_p) = q(D_e/L_e)[\cosh(w_p - x)/\sinh(w_p - x_p)](n_i^2/N_{Ap})(e^{qv_{AB}/kT} - 1)
  \]
  \[
  J_h(x_n < x < w_n) = q(D_h/L_h)[\cosh(w_n - x)/\sinh(w_n - x_n)](n_i^2/N_{Dn})(e^{qv_{AB}/kT} - 1)
  \]

- **Total current is found from continuity across SCL:**
  \[
  i_D(v_{AB}) = A [J_e(-x_p) + J_h(x_n)] = I_S (e^{qv_{AB}/kT} - 1), \text{ with } I_S = A q n_i^2 [(D_h/N_{Dn} w_n^*) + (D_e/N_{Ap} w_p^*)]
  \]

  Note: $w_p^*$ and $w_n^*$ are the effective widths of the p- and n-sides
  If $L_e >> w_p$, then $w_p^* \approx (w_p - x_p)$, and if $L_e << w_p$, then $w_p^* \approx L_e$
  If $L_h >> w_n$, then $w_n^* \approx (w_n - x_n)$, and if $L_h << w_n$, then $w_n^* \approx L_h$

- **Features and limitations of the model**
  Exponential dependence enters via boundary conditions
  Injection is predominantly into more lightly doped side
  Saturation current, $I_S$, goes down as doping levels go up

  **Limits:**
  1. SCL g-r may dominate at low current levels
  2. Series resistance may reduce junction voltage at high currents
  3. Junction may breakdown (conduct) at large reverse bias
6.012 Microelectronic Devices and Circuits
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