Lecture 5
PN Junction and MOS
Electrostatics(II)

PN JUNCTION IN THERMAL EQUILIBRIUM

Outline

1. Introduction
2. Electrostatics of pn junction in thermal equilibrium
3. The depletion approximation
4. Contact potentials

Reading Assignment:
Howe and Sodini, Chapter 3, Sections 3.3-3.6
1. Introduction

• pn junction
  – p-region and n-region in intimate contact

Why is the p-n junction worth studying?

It is present in virtually every semiconductor device!

Example: CMOS cross-section

Understanding the pn junction is essential to understanding transistor operation
2. Electrostatics of p-n junction in equilibrium

Focus on intrinsic region:

Doping distribution of an **abrupt** p-n junction
What is the carrier concentration distribution in thermal equilibrium?

First think of the two sides separately:

Now bring the two sides together.

What happens?
Resulting carrier concentration profile in thermal equilibrium:

- Far away from the metallurgical junction: nothing happens
  - Two *quasi-neutral regions*
- Around the metallurgical junction: diffusion of carriers must counter-balance drift
  - *Space-charge region*
On a linear scale:

**Thermal equilibrium:** balance between drift and diffusion

\[
J_n(x) = J_n^{\text{drift}}(x) + J_n^{\text{diff}}(x) = 0
\]

\[
J_p(x) = J_p^{\text{drift}}(x) + J_p^{\text{diff}}(x) = 0
\]

We can divide semiconductor into three regions

- Two quasi-neutral n- and p-regions (QNR’s)
- One space-charge region (SCR)

Now, we want to know \(n_o(x), p_o(x), \rho(x), E(x)\) and \(\phi(x)\).

**We need to solve Poisson’s equation using a simple but powerful approximation**
3. The Depletion Approximation

- Assume the QNR’s are perfectly **charge neutral**
- Assume the SCR is **depleted** of carriers
  - *depletion region*
- Transition between SCR and QNR’s sharp at
  - $-x_{no}$ and $x_{no}$ (**must calculate where to place these**)

\[ x < -x_{po}; \quad p_o(x) = N_a, \quad n_o(x) = \frac{n_i}{N_a} \]
\[ -x_{po} < x < 0; \quad p_o(x), \quad n_o(x) \ll N_a \]
\[ 0 < x < x_{no}; \quad n_o(x), \quad p_o(x) \ll N_d \]
\[ x > x_{no}; \quad n_o(x) = N_d, \quad p_o(x) = \frac{n_i^2}{N_d} \]
Space Charge Density

\[
\rho(x) = 0; \quad x < -x_{po}
\]
\[
= -qN_a; \quad -x_{po} < x < 0
\]
\[
= qN_d; \quad 0 < x < x_{no}
\]
\[
= 0; \quad x > x_{no}
\]
Electric Field

Integrate Poisson’s equation

\[ E(x_2) - E(x_1) = \frac{1}{\varepsilon_s} \int_{x_1}^{x_2} \rho(x) \, dx \]

\[ x < -x_{po}; \quad E(x) = 0 \]

\[ -x_{po} < x < 0; \quad E(x) - E(-x_{po}) = \frac{1}{\varepsilon_s} \int_{-x_{po}}^{x} -qN_a \, dx' \]

\[ = \left[ -\frac{qN_a}{\varepsilon_s} x \right]_{-x_{po}}^{x} = \frac{-qN_a}{\varepsilon_s} (x + x_{po}) \]

\[ 0 < x < x_{no}; \quad E(x) = \frac{qNa}{\varepsilon_s} (x - x_{no}) \]

\[ x > x_{no}; \quad E(x) = 0 \]
Electrostatic Potential
(with $\phi=0$ @ $n_o=p_o=n_i$)

$$\phi = \frac{kT}{q} \cdot \ln \frac{n_o}{n_i} \quad \phi = -\frac{kT}{q} \cdot \ln \frac{p_o}{n_i}$$

In QNRs, $n_o$ and $p_o$ are known $\Rightarrow$ can determine $\phi$

in p-QNR: $p_o=N_a \Rightarrow \phi_p = -\frac{kT}{q} \cdot \ln \frac{N_a}{n_i}$

in n-QNR: $n_o=N_d \Rightarrow \phi_n = \frac{kT}{q} \cdot \ln \frac{N_d}{n_i}$

Built-in potential:

$$\phi_B = \phi_n - \phi_p = \frac{kT}{q} \cdot \ln \frac{N_d N_a}{n_i^2}$$

This expression is always correct in TE!
We did not use depletion approximation.
To obtain $\phi(x)$ in between, integrate $E(x)$

$$\phi(x_2) - \phi(x_1) = - \int_{x_1}^{x_2} E(x') \, dx'$$

$x < -x_{po}$; \hspace{1cm} $\phi(x) = \phi_p$

$-x_{po} < x < 0$; \hspace{1cm} $\phi(x) - \phi(-x_{po}) = - \int_{-x_{po}}^{x} \frac{qNa}{\varepsilon_s} (x' + x_{po}) \, dx'$

$$= \frac{qNa}{2\varepsilon_s} \left(x + x_{po}\right)^2$$

$$\phi(x) = \phi_p + \frac{qNa}{2\varepsilon_s} \left(x + x_{po}\right)^2$$

$0 < x < x_{no}$; \hspace{1cm} $\phi(x) = \phi_n - \frac{qNd}{2\varepsilon_s} (x - x_{no})^2$

$x > x_{no}$; \hspace{1cm} $\phi(x) = \phi_n$

Almost done ....
Still do not know $x_{no}$ and $x_{po} \Rightarrow$ need two more equations

1. Require overall charge neutrality:

$$qN_a x_{po} = qN_d x_{no}$$

2. Require $\phi(x)$ to be continuous at $x=0$;

$$\phi_p + \frac{qN_a}{2\varepsilon_s} x_{po}^2 = \phi_n - \frac{qN_d}{2\varepsilon_s} x_{no}^2$$

Two equations with two unknowns — obtain solution:

$$x_{no} = \sqrt{\frac{2\varepsilon_s \phi_B N_a}{q(N_a + N_d)N_d}}$$  \hspace{1cm}  x_{po} = \sqrt{\frac{2\varepsilon_s \phi_B N_d}{q(N_a + N_d)N_a}}$$

Now problem is completely solved!
Solution Summary

Space Charge Density

Electrostatic Field

Electrostatic Potential
Other results:

Width of the space charge region:

\[ x_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_B (N_a + N_d)}{qN_aN_d}} \]

Field at the metallurgical junction:

\[ |E_o| = \sqrt{\frac{2q\phi_B N_a N_d}{\varepsilon_s (N_a + N_d)}} \]
Three Special Cases

- Symmetric junction: \( N_a = N_d \)
  \[ x_{po} = x_{no} \]

- Asymmetric junction: \( N_a > N_d \)
  \[ x_{po} < x_{no} \]

- Strongly asymmetric junction
  - p\(^+\)n junction: \( N_a >> N_d \)
    \[ x_{po} \ll x_{no} \approx x_{do} \approx \sqrt{\frac{2 \varepsilon_s \phi_B}{qN_d}} \]
    \[ |E_o| \approx \sqrt{\frac{2q\phi_B N_d}{\varepsilon_s}} \]

The lightly-doped side controls the electrostatics of the pn junction
4. Contact Potential

Potential distribution in thermal equilibrium so far:

**Question 1:** If I apply a voltmeter across the pn junction diode, do I measure $\phi_B$?

- [ ] yes
- [ ] no
- [ ] it depends

**Question 2:** If I short terminals of pn junction diode, does current flow on the outside circuit?

- [ ] yes
- [ ] no
- [ ] sometimes
We are missing contact potential at the metal-semiconductor contacts:

**Metal-semiconductor contacts:** junction of dissimilar materials
⇒ built-in potentials at contacts $\phi_{mn}$ and $\phi_{mp}$.

Potential difference across structure must be zero
⇒ Cannot measure $\phi_B$.

$$\phi_B = |\phi_{mn}| + |\phi_{mp}|$$
5. PN Junction-Reverse Bias

Assume: No Current Flows

\[ \phi_j = \phi_B - V_D \]

Same Analysis applies:

Substitute

\[ x_{do} = x_{po} + x_{no} = \sqrt{\frac{2 \varepsilon_s (\phi_B - V_D)(N_a + N_d)}{qN_aN_d}} \]
What did we learn today?

**Summary of Key Concepts**

- Electrostatics of pn junction in equilibrium
  - A *space-charge region* surrounded by two *quasi-neutral regions* formed.
- To first order, carrier concentrations in space-charge region are much smaller than the doping level
  - ⇒ can use *Depletion Approximation*
- From contact to contact, there is no potential build-up across the pn junction diode
  - Contact potential(s).
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