Lecture 16
The pn Junction Diode (III)

Outline

• I-V Characteristics (Review)
• Small-signal equivalent circuit model
• Carrier charge storage
  –Diffusion capacitance

Reading Assignment:
Howe and Sodini; Chapter 6, Sections 6.4 - 6.5
1. I-V Characteristics (Review)

Diode Current Equation:

\[ I_D = I_o \left[ e^{\left(\frac{V_D}{V_{th}}\right)} - 1 \right] \]
Physics of forward bias:

Diode Current equation:

\[ I_D = I_o \left[ \exp \left( \frac{qV_D}{kT} \right) - 1 \right] \]

- Junction potential \( \phi_j \) (potential drop across SCR) reduced by \( |V_D| \)
  \[ \Rightarrow \text{minority carrier injection into QNRs} \]
- Minority carrier diffusion through QNRs
- Minority carrier recombination at contacts to the QNRs (and surfaces)
- Large supply of carriers injected into the QNRs
  \[ \Rightarrow I_D \propto \exp \left[ \frac{qV_D}{kT} \right] \]
Physics of reverse bias:

\[ I_D = I_o \left[ \exp \left( \frac{qV_D}{kT} \right) - 1 \right] \]

- Junction potential \( \phi_j \) (potential drop across SCR) increased by \( |V_D| \)
  - \( \Rightarrow \) **minority carrier extraction** from QNRs
- Minority carrier diffusion through QNRs
- Minority carrier generation at surfaces & contacts of QNRs
- Very small supply of carriers available for extraction
  - \( \Rightarrow I_D \) saturates to small value
  - \( \Rightarrow I_D \approx -I_o \)
2. Small-signal equivalent circuit model

Examine effect of small signal overlapping bias:

\[ i_D = I_D + i_d = I_o \left[ \exp\left( \frac{q(V_D + v_d)}{kT} \right) - 1 \right] \]

If \( v_d \) small enough, linearize exponential characteristics:

\[ I_D + i_d = I_o \left[ \exp\left( \frac{qV_D}{kT} \right) \exp\left( \frac{qv_d}{kT} \right) - 1 \right] \]

\[ = I_o \left[ \exp\left( \frac{qV_D}{kT} \right) \left( 1 + \frac{qv_d}{kT} \right) - 1 \right] \]

\[ = I_o \left[ \exp\left( \frac{qV_D}{kT} \right) - 1 \right] + I_o \exp\left( \frac{qV_D}{kT} \right) \frac{qv_d}{kT} \]

Then:

\[ i_d = \frac{q(I_D + I_o)}{kT} \cdot v_d \]

From a small signal point of view, Diode behaves as conductance of value:

\[ g_d = \frac{q(I_D + I_o)}{kT} \approx \frac{qI_D}{kT} \]
Small-signal equivalent circuit model

\[ g_d = \frac{qI_D}{kT} \]

g_d depends on bias. In forward bias, \( g_d \) is linear in diode current.
Capacitance associated with depletion region:

Depletion or junction capacitance:

\[ C_j = C_j(V_D) = \frac{dq_J}{dV_D} \bigg|_{V_D} \]

\[ C_j = A \sqrt{\frac{q \varepsilon_s N_a N_d}{2(N_a + N_d)(\phi_B - V_D)}} \]
Small-signal equivalent circuit model

\[
C_j = A \sqrt{\frac{q \varepsilon_s N_a N_d}{2(N_a + N_d) \phi_B}} \cdot \sqrt{\frac{\phi_B}{\phi_B - V_D}}
\]

or,

\[
C_j = \frac{C_{jo}}{\sqrt{1 - \frac{V_D}{\phi_B}}}
\]

Under Forward Bias assume 

\[
V_D \approx \frac{\phi_B}{2}
\]

\[
C_j = \sqrt{2} C_{jo}
\]

\[C_{jo} \equiv \text{zero-voltage junction capacitance}\]
3. Charge Carrier Storage: diffusion capacitance

What happens to majority carriers?

Carrier picture thus far:

If QNR minority carrier concentration ↑ but majority carrier concentration unchanged? ⇒ quasi-neutrality is violated.
Quasi-neutrality demands that at every point in QNR:

**excess minority carrier concentration**

= **excess majority carrier concentration**

In n-type Si, at every x:

\[ p_n(x) - p_{no} = n_n(x) - n_{no} \]

In p-type Si, at every x:

\[ n_p(x) - n_{po} = p_p(x) - p_{po} \]
Quasi-neutrality demands that at every point in QNR:

*excess minority carrier concentration*

= *excess majority carrier concentration*

Mathematically:

\[ p'_n(x) = p_n(x) - p_{no} \approx n'_n(x) = n_n(x) - n_{no} \]

Define integrated carrier charge:

\[ q_{pn} = qA \frac{1}{2} \int_{x_n}^{W_n} p'(x_n) \cdot (W_n - x_n) \, dx_n \]

\[ = qA \frac{W_n - x_n}{2} \frac{n_i^2}{N_d} \exp \left[ \frac{qV_D}{kT} - 1 \right] = -q_{Nn} \]
Now examine small increase in $V_D$:

Small increase in $V_D \Rightarrow$ small increase in $q_{Pn} \Rightarrow$ small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$C_{dn} = \left. \frac{dq_{Pn}}{dv_D} \right|_{v_D = V_D} = qA \frac{W_n - x_n}{2} \frac{n_i^2}{N_d} \frac{q}{kT} \exp \left[ \frac{qV_D}{kT} \right]$$
Can write in terms of $I_{Dp}$ (portion of diode current due to holes in n-QNR):

$$C_{dn} = \frac{q}{kT} \frac{(W_n - x_n)^2}{2D_p} qA \frac{n_i^2}{N_d} \frac{D_p}{W_n - x_n} \exp\left[\frac{qV_p}{kT}\right]$$

$$\approx \frac{q}{kT} \frac{(W_n - x_n)^2}{2D_p} I_{Dp}$$

Define *transit time* of holes through n-QNR:

$$\tau_{Tp} = \frac{(W_n - x_n)^2}{2D_p}$$

Transit time is the *average time for a hole to diffuse through n-QNR* [will discuss in more detail in BJT]

Then:

$$C_{dn} \approx \frac{q}{kT} \cdot \tau_{Tp} \cdot I_{Dp}$$
Similarly for p-QNR:

\[ C_{dp} \approx \frac{q}{kT} \cdot \tau_{Tn} \cdot I_{Dn} \]

where \( \tau_{Tn} \) is *transit time* of electrons through p-QNR:

\[ \tau_{Tn} = \frac{(W_p - x_p)^2}{2D_n} \]

Both capacitors sit in *parallel* \( \Rightarrow \) total diffusion capacitance:

\[ C_d = C_{dn} + C_{dp} = \frac{q}{kT} (\tau_{Tn}I_{Dn} + \tau_{Tp}I_{Dp}) \]

**Complete small-signal equivalent circuit model for diode:**

![Circuit Diagram](https://via.placeholder.com/150)
Bias dependence of $C_j$ and $C_d$:

- $C_j$ dominates in reverse bias and small forward bias
  \[ \propto \frac{1}{\sqrt{\phi_B - V_D}} \]

- $C_d$ dominates in strong forward bias
  \[ \propto \exp\left[ \frac{qV_D}{kT} \right] \]
What did we learn today?

Summary of Key Concepts

Large and Small-signal behavior of diode:

- **Diode Current:**
  \[ I = I_o \left( e^{\frac{qV_D}{kT}} - 1 \right) \]

- **Conductance:** associated with current-voltage characteristics
  - \( g_d \propto I \) in forward bias,
  - \( g_d \) negligible in reverse bias

- **Junction capacitance:** associated with charge modulation in depletion region
  \[ C_j \propto \frac{1}{\sqrt{\phi_B - V_D}} \]

- **Diffusion capacitance:** associated with charge storage in QNRs to maintain quasi-neutrality.
  \[ C_d \propto e^{\frac{qV_D}{kT}} \]