Recitation 3: Carrier Action

Yesterday we talked about the movement of the carriers inside a semiconductor. There is a direct relationship between the velocity of carriers and the electrical current that is generated.

\[
\text{Current Density } = |J_n| = |n \cdot q \cdot v_n| \\
= |J_p| = |p \cdot q \cdot v_p|
\]

This is because:

\[
|I| = \left| \frac{\# \text{ of charges across cross-section area}}{\text{time}} \right| = \left| \frac{Q}{t} \right| = \left| \frac{q \cdot \# \text{ of charges across cross-section area}}{t} \right|
\]

\[
= \left| \frac{q \cdot \# \text{ density} \cdot \text{volume}}{t} \right| = \left| \frac{q \cdot n \cdot L \cdot A}{t} \right| = |q \cdot n \cdot v_n \cdot A| \cdot \frac{L}{t} = \text{velocity}
\]

\[
|I_n| = |J_n| = |q \cdot n \cdot v_n|
\]

<table>
<thead>
<tr>
<th>Table 1: Drift vs. Diffusion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Drift</strong></td>
</tr>
<tr>
<td>Due to electric field $E$</td>
</tr>
<tr>
<td>$\frac{dn}{dx}$</td>
</tr>
<tr>
<td>$v_n = \mu_n E$</td>
</tr>
<tr>
<td>$J_n = -q \cdot n \cdot v_n = q \cdot n \cdot \mu_n \cdot E$</td>
</tr>
<tr>
<td>$J_p = q \cdot p \cdot v_p = q \cdot p \cdot \mu_p \cdot E$</td>
</tr>
<tr>
<td><strong>Important Parameter</strong></td>
</tr>
<tr>
<td>$\mu_n = \frac{q \cdot \tau_c}{2 \cdot m_n}$</td>
</tr>
<tr>
<td>$D_n = \frac{kT}{\mu n}$</td>
</tr>
</tbody>
</table>
Note: ** physical intuition rather than remembering the equation **

- Current (can usually measure) always related to charge velocity (can back calculate)
- $\tau_c$ (collision time) is related to the **imperfection** of the lattice
- Mobility depends on collision time and temperature:
  1. $\mu \propto \tau_c$: doping (impurity) increases $\Rightarrow$ more collisions $\Rightarrow$ $\mu \downarrow$
  2. temperature (lattice vibration): higher $T$ $\Rightarrow$ more collision $\Rightarrow$ $\mu \downarrow$
- same semiconductor, the difference between $\mu_n$ & $\mu_p$ are due to $m_n$ & $m_p$
- high mobility is extremely important for high performance devices
  
  Si: $\mu_n = 1400 \text{ cm}^2/\text{V sec}$ $\mu_p = 500 \text{ cm}^2/\text{V sec}$ for doping $10^{13} \text{ cm}^{-3}$
  GaAs: $\mu_n = 8000 \text{ cm}^2/\text{V sec}$ $\mu_p = 400 \text{ cm}^2/\text{V sec}$

![Diagram of carrier mobility vs. doping concentration in Si](image-url)

**Carrier Mobility vs. Doping Concentration in Si**
Example 1: Integrated Resistor

Our first IC device:

\[ J = J_n + J_p = q(n\mu_n + p\mu_p)E \]

\[ E = \frac{V}{L}, \quad A = w \times t \]

\[ I = J \cdot A = \left( q(n\mu_n + p\mu_p) \frac{V}{L} \right) \cdot (w \times t) \]

\[ I = \left[ q(n\mu_n + p\mu_p) \frac{w \times t}{L} \right] \cdot V \]

But \[ I = \frac{V}{R} \text{ Ohm's Law} \]

\[ \therefore R = \frac{1}{q(n\mu_n + p\mu_p) \frac{w \times t}{L}} = \frac{1}{q(n\mu_n + p\mu_p)} \cdot \frac{L}{w \times t} = \rho \cdot \frac{L}{w \times t} \]
Resistivity = \( \rho = \frac{1}{q(n\mu_n + p\mu_p)} \) or \( \sigma = q(n\mu_n + p\mu_p) \)

Usually majority dominates resistivity (n-type majority \( \Rightarrow \rho \approx \frac{1}{q \cdot n \cdot \mu_n} \), and vice versa).

Since \( \rho \) (or \( \sigma \)) can be measured easily, it can be used to derive doping of a semiconductor (n or p). If we take a Si wafer, it will be hard to know the doping \( a \ priori \) unless someone specifies the doping level, but we can use resistivity to find out.

**Example 2: Resistivity of Si**

What is the resistivity of (1) intrinsic Si, (2) Si with \( N_d = 10^{13} \) and (3) Si with \( N_a = 10^{20} \)?

1. \( n_o = p_o = 10^{10} \text{ cm}^{-3} \). Therefore, \( \rho \) is:

\[
\rho = \frac{1}{1.6 \times 10^{-19} \cdot (1450 \text{ cm}^2/\text{V} \cdot \text{sec} \times 10^{10} \text{ cm}^{-3} + 500 \text{ cm}^2/\text{V} \cdot \text{sec} \times 10^{10} \text{ cm}^{-3})} = 1.6 \times 10^{-19} \times 1.95 \times 10^{13} = 3.2 \times 10^5 \Omega \cdot \text{cm} \text{ (make sure the units are correct)}
\]

Poor conductivity, quite insulating

2. \( N_d = 10^{13} \text{ cm}^{-3} \gg n_i = 10^{10} \Rightarrow n_o \approx N_d = 10^{13} \), \( p_o = \frac{n_i^2}{n_o} = 10^7 \)

\[
\rho \approx \frac{1}{q \cdot n \cdot \mu_n + p \cdot \mu_p} = \frac{1}{1.6 \times 10^{-19} \times (1450 \times 10^{13} + 500 \times 10^7)} = 430 \Omega \cdot \text{cm}
\]

(check on the curve)

3. \( N_a = 10^{20} \gg n_i = 10^{10} \), \( p_o \approx N_a = 10^{20} \), \( n_o \approx \frac{n_i^2}{p_o} = 1 \)

\[
\rho \approx \frac{1}{q \cdot p \cdot \mu_p} = \frac{1}{1.6 \times 10^{-19} \times 50 \times 10^{20}} = 1.25 \times 10^{-3} \Omega \cdot \text{cm like metal}
\]

From this example, we can see that Si resistivity can be tuned several orders of magnitude by doping, from insulator-like to metal-like.
Sheet Resistance

\[ R = \left( \frac{\rho}{t} \right) \left( \frac{L}{w} \right) \]

The unit of \( \rho \) is \( \Omega \cdot \text{cm} \), the unit of \( t \) is cm meaning that the unit of \( \left( \frac{\rho}{t} \right) \) is \( \Omega \) - that of resistance. We call \( \left( \frac{\rho}{t} \right) \) the sheet resistance \( R_s \). This is a convenient metric for IC design as:

- \( \rho, t \): process and material parameters
- \( \frac{L}{w} \): # of squares with dimensions \( w \) - layout design parameter

Sheet resistance is also a very useful parameter to characterize (thin) film resistivity.

Fabricating an IC Resistor

How to fabricate an IC resistor?

Make an n-type region in a p-type substrate. We will see why this isolation can work soon.