Recitation 4: Electrostatic Potential & Carrier Concentration

Yesterday in lecture, we learned that under T.E. (thermal equilibrium), there is a fundamental relationship between the electrostatic potential $\phi(x)$, at one location $(x)$ within the semiconductor and the carrier concentration at that location.

$$
\phi(x) = \frac{kT}{q} \ln \frac{n_o(x)}{n_i} = -\frac{kT}{q} \ln \frac{p_o(x)}{n_i}
$$

By the Boltzman relationship (or the 60 mV rule),

$$
\phi(x) = (60 \text{ mV}) \ln \frac{n_o(x)}{n_i} = -(60 \text{ mV}) \ln \frac{p_o(x)}{n_i}
$$

How did this relation come about?

Revisit Thermal Equilibrium

1. Under T.E. can we have electrostatic field (or voltage) within the semiconductor?
   Yes. (we do not have “external” energy source). There can be static electrostatic field inside the semiconductor generated by “space” charges.

2. Under T.E. can we have an overall current?
   No. That will give rise to charge piling up.

Some Fundamentals about Electrostatics

Relationship between electrostatic potential $\phi(x)$, electric field $E(x)$, and (space) charge density $\rho(x)$:

1. First, the charge density we are talking about here is the “Net” charge density, we call space charge density.

   in n-type material: $\rho(x) = q(N_d(x) - n(x))$
   p-type material: $\rho(x) = q(p(x) - N_a(x))$

   ($N_d, N_a$ are space charges. $\rho(x)$ is the extra which can not be compensated by $e^-$ or hole free charges) $\rightarrow$ space charge density.

2. $\frac{dE}{dx} = \frac{\rho}{\varepsilon}$ ($\varepsilon =$ electric permittivity Farad/cm). In other words,

   $$
   E(x) - E(0) = \frac{1}{\varepsilon} \int_0^x e(x') \, dx'
   $$
Also, \( \frac{d\phi}{dx} = -E(x) \). In other words:

\[
\phi(x) - \phi(0) = -\int_0^x E(x') \, dx'
\]

The two equations above can be combined to give the following relation:

\[
\frac{d^2\phi(x)}{dx^2} = -\frac{dE(x)}{dx} = -\frac{\rho(x)}{\epsilon}
\]

**Boltzman Relation**

Basically the Boltzman relationship exists due to thermal equilibrium. Under T.E., for n-type:

\[
J_n = 0
\]

But \( J_n = q \cdot n_o \cdot \mu_n \cdot E + q \cdot D_n \cdot \frac{dn_o}{dx} = 0 \)

\[
-q \cdot n_o \cdot \mu_n \cdot \frac{d\phi}{dx} = -q \cdot D_n \cdot \frac{dn_o}{dx}
\]

\[
\frac{\mu_n}{D_n} \frac{d\phi}{dx} = \frac{1}{n_o} \frac{dn_o}{dx}
\]

\[
\frac{q}{k \cdot T} \frac{d\phi}{dx} = \frac{d(\ln(n_o))}{dx}
\]

Integrate:

\[
\frac{q}{k \cdot T} (\phi - \phi_{ref}) = \ln(n_o) - \ln(n_{o,ref}) = \ln \left( \frac{n_o}{n_{o,ref}} \right)
\]

Similarly, for p-type:

\[
J_p = 0
\]

But \( J_p = q \cdot p_o \cdot \mu_p \cdot E - q \cdot D_p \cdot \frac{dp_o}{dx} = 0 \)

\[
-q \cdot p_o \cdot \mu_p \cdot \frac{d\phi}{dx} = q \cdot D_p \cdot \frac{dp_o}{dx}
\]

\[
\frac{\mu_p}{D_p} \frac{d\phi}{dx} = \frac{1}{p_o} \frac{dp_o}{dx}
\]

\[
\frac{q}{k \cdot T} \frac{d\phi}{dx} = \frac{d(\ln(p_o))}{dx}
\]

Integrate:

\[
-\frac{q}{k \cdot T} (\phi - \phi_{ref}) = \ln(p_o) - \ln(p_{o,ref}) = \ln \left( \frac{p_o}{p_{o,ref}} \right)
\]

Set \( \phi_{ref} = 0 \) at \( n_{o, ref} = p_{o, ref} = n_i \). Then:

\[
-\frac{q}{k \cdot T} \phi = \ln \left( \frac{p_o}{n_i} \right)
\]

\[
\phi = -\frac{k \cdot T}{q} \ln \left( \frac{p_o}{n_i} \right)
\]

or

\[
p_o = n_i e^{-\frac{q\phi}{k \cdot T}}
\]
Example

Now let us look at a particular example. We have a doping profile \( N_d(x) = N_{do} + \Delta N_d(1 - e^{-\frac{x}{L_c}}) \). \( N_{do} = 10^{16} \text{ cm}^{-3} \), \( \Delta N_d = 9 \times 10^{16} \text{ cm}^{-3} \), \( L_c = 10 \mu m \). We would like to know:

1. What is the electrostatic profile \( \phi(x) \)?
2. How about electric field \( E(x) \)?
3. Space charge density \( \rho(x) \)?

First, we have \( \phi(x) \) vs. \( n_o(x), p_o(x) \) from the Boltzmann relationships. If we know \( n_o(x) \), or \( p_o \), we can find \( \phi(x) \).

But does \( n_o(x) = N_d(x) \)? In reality, it should not, if \( n_o(x) = N_d(x) \), we will have a net current due to diffusion Not T.E. anymore.

To obtain an accurate solution, we have:

\[
J_n = q n_o \mu_n E + q n_o \frac{d n_o}{dx} = 0 \quad (N_d \text{ doping, electron majority carrier, only consider } J_n \text{ here.})
\]

\[
\frac{dE}{dx} = \frac{q}{\epsilon_{Si}} (N_d - n_o)
\]

With these two, we get:

\[
E = -\frac{D_n}{\mu_n} n_o \frac{d n_o}{dx} \Rightarrow k \cdot T \frac{d^2 (\ln n_o)}{dx^2} = \frac{1}{\epsilon_{Si}} (n_o(x) - N_d)
\]

But very hard to solve \( n_o(x) \). In most cases, an analytical solution is impossible. Can we do something simpler?

We make approximations! The first type of scenario is \( n_o(x) \simeq N_d \) ("quasi-neutrality"). If we assume \( n_o(x) \simeq N_d = N_{do} + \Delta N_d(1 - e^{-\frac{L_c}{x}}) \):

Define \( a \triangleq N_{do} + \Delta N_d(1 - e^{-\frac{L_c}{x}}) \)

\[
\phi(x) = \frac{k \cdot T}{q} \ln \frac{n_o(x)}{n_i} \simeq \frac{k \cdot T}{q} \ln \frac{a}{n_i} \frac{x}{L_c}
\]

\[
E(x) = -\frac{d\phi(x)}{dx} \simeq -\frac{k \cdot T}{q} \frac{1}{a} \Delta N_d \frac{1}{L_c} e^{-\frac{x}{L_c}}
\]

\[
\rho(x) = \epsilon_{Si} \frac{dE(x)}{dx} \simeq \epsilon_{Si} \frac{k \cdot T}{q} \left( \frac{1}{a^2} \Delta N_d^2 \frac{1}{L_c^2} e^{-\frac{2x}{L_c}} + \frac{1}{a} \Delta N_d \frac{1}{L_c^2} e^{-\frac{x}{L_c}} \right)
\]

\[
\simeq \epsilon_{Si} \frac{k \cdot T}{q} \frac{1}{a^2} \left( \frac{\Delta N_d (N_{do} + \Delta N_d)}{L_c^2} \right) e^{-\frac{x}{L_c}}
\]
To satisfy quasi-neutrality, we need:

\[
\left| \frac{n_0(x) - N_d(x)}{N_d(x)} \right| \ll 1, \text{ we know } (n_0(x) - N_d(x)) = -\frac{\rho(x)}{q} \\
\left| \frac{n_0(x) - N_d(x)}{N_d(x)} \right| = \left| \frac{\rho(x)}{q \cdot N_d(x)} \right| \approx \frac{\epsilon_{Si} k \cdot T \Delta N_d(N_{do} + \Delta N_d)}{a^3} \frac{1}{L_c^2} e^{-\frac{x}{L_c}} \\
\left| \frac{n_0(x) - N_d(x)}{N_d(x)} \right| < \epsilon_{Si} \frac{k \cdot T \Delta N_d(N_{do} + \Delta N_d)}{a^3} \frac{1}{(N_{do})^3} \frac{1}{L_c^2} = 1.46 \times 10^{-4} \ll 1
\]

Therefore, our quasi-neutrality is valid. This quasi-neutrality satisfies when doping profile is gradual. If we have time, we can verify \( J_n^{\text{diff}} = q \cdot D_n \frac{dn_0(x)}{dx} = J_n^{\text{drift}} = q \mu_n n_0(x) E(x) \) using the above equations.