Recitation 18: BJT-Regions of Operation & Small Signal Model

BJT: Regions of Operation

System of equations that describes BJT operation:

\[
I_c = I_s \left(\exp\left(\frac{qV_{BE}}{kT}\right) - \exp\left(\frac{qV_{BC}}{kT}\right)\right) - \frac{I_s}{\beta_R} \left(\exp\left(\frac{qV_{BC}}{kT}\right) - \exp\left(\frac{qV_{BE}}{kT}\right) - 1\right)
\]

\[
I_B = \frac{I_s}{\beta_F} \left(\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right) + \frac{I_s}{\beta_R} \left(\exp\left(\frac{qV_{BC}}{kT}\right) - 1\right)
\]

\[
I_E = -I_s \left(\exp\left(\frac{qV_{BE}}{kT}\right) - \exp\left(\frac{qV_{BC}}{kT}\right)\right) - \frac{I_s}{\beta_F} \left(\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right)
\]

\[
I_s = \frac{qA_E n_i^2 D_{nB}}{N_{nB} w_B}
\]

\[
\beta_F = \frac{N_{DF} D_{nB} w_E}{N_{nB} D_{pE} w_B}
\]

\[
\beta_R = \frac{N_{DC} D_{nB} w_C}{N_{nB} D_{pC} w_B}
\]

This set of equations can describe all four regimes of operation for BJT

**Forward Active:** \(V_{BE} > 0, V_{BC} < 0\)
Reverse Active (RAR)

\( V_{BE} < 0, V_{BC} > 0 \)

![Diagram of Reverse Active Region]

Cut-off

\( V_{BE} < 0, V_{BC} < 0 \)

![Diagram of Cut-off Region]

Saturation

\( V_{BE} > 0, V_{BC} > 0 \)

![Diagram of Saturation Region]
Understanding the $I_C$ vs. $V_{CE}$ curve: $I_C$ drops rapidly below $V_{CE,SAT} \approx 0.1$ to 0.2 V.

Why?

- Each curve $I_B$ is fixed
- $V_{CE} = V_{BE} - V_{BC}$, $\implies V_{BC} = V_{BE} - V_{CE}$
- When $V_{CE}$ is large, $V_{BC} < 0$, FAR. As we reduce $V_{CE}$, $V_{BC}$ reduces, at some point, $V_{BC}$ starts to become forward biased. Now, hole flux from B $\rightarrow$ C increases exponentially from Law of Junction; to keep $I_B$ constant, hole flux into emitter must be reduced, $\implies V_{BE}$ drops, $\implies I_C$ drops quickly.

Small Signal Model of BJT

(Next week we will be using BJT & MOSFET for amplifier circuits) Want to know the small signal circuit model of BJT

1. Transconductance $g_m = \frac{\delta i_c}{\delta V_{BE}} |_Q$

\[ I_c = I_s e^{qV_{BE}/kT} \implies g_m = \frac{q}{kT} I_s e^{qV_{BE}/kT} = \frac{I_c}{V_{th}} \]

Note, different from MOSFET: $g_m \simeq \sqrt{\frac{W}{L}} I_D$ (depends upon device size), but not for bipolar case.
2. Input resistance:

\[
I_B = \frac{I_s e^{qV_{BE}/kT}}{\beta_F}
\]

\[
g_\pi = \frac{1}{\gamma_\pi} = \frac{\delta i_B}{\delta V_{BE}} = \frac{I_B}{V_{th}} = \frac{g_m}{\beta_F}
\]

or \( \gamma_\pi = \frac{\beta_F}{g_m} \)

The input resistance of MOSFET is \( \infty \). In order to have a high input resistance for BJT, need high current gain \( \beta_F \).

Example: npn with \( \beta_F = 150, I_c = \text{mA} \)

\[
g_m = \frac{I_c}{V_{th}} = \frac{1 \times 10^{-3} \text{A}}{0.025 \text{V}} = 40 \text{mS}
\]

\[
g_\pi = \frac{g_m}{\beta_F} = \frac{40 \text{mS}}{150} = 0.267 \text{mS} (\gamma_\pi = 3.7 \text{k}\Omega)
\]

3. Output resistance: Ebers-Moll model have perfect current source in FAR. Real characteristics show some increase in \( i_c \) with \( V_{CE} \)

\[
g_o = \frac{\delta i_c}{\delta V_{CE}} \text{ where does } g_o \text{ come from?}
\]

In FAR: \( I_c = I_s e^{qV_{BE}/kT} = \frac{qA_e^2n^2D_nB e^{qV_{BE}/kT}}{N_{aB}w_B} \)

\( w_B \) shrinks as \( |V_{BC}| \uparrow \), thus \( I_c \uparrow \).

Model: \( g_o = \text{slope} = \frac{I_c}{V_{CE} + V_A} \cong \frac{I_c}{V_A} (V_A \gg V_{CE}) \)

\[
g_o = \frac{1}{\gamma_o} = \frac{I_c}{V_A}
\]

Example: \( I_c = 100 \mu\text{A}, V_A = 35 \text{V}, \Rightarrow \gamma_o = 350 \text{k}\Omega \)

\( V_A \) increases with increasing base width and increasing base doping. This is also why \( N_{aB} \) usually \( \gg N_{aC} \)
Now what do we have so far? Need to add capacitances...

\[
\begin{align*}
\text{(B-E): } C_{JE} &= \sqrt{\frac{q\epsilon_s N_a B N_{dE}}{2(N_{aB} + N_{dE})(\phi_{BE} - V_{BE})}} \quad (\because N_{dE} \gg N_{aB}) \\
\text{(B-E): } C_{JC} &= \sqrt{\frac{q\epsilon_s N_a B N_{dC}}{2(N_{aB} + N_{dC})(\phi_{BC} - V_{BC})}} \approx \sqrt{\frac{q\epsilon_s N_{dC}}{2(\phi_{BC} - V_{BE})}}
\end{align*}
\]

- Both are functions of bias
- Since \( N_{aB} \gg N_{dC} \), \( C_{JE} \gg C_{JC} \). \( C_{JC} \) is often called \( C_\mu \).

**Diffusion Capacitance**

\[
\begin{align*}
C_b &= \frac{\delta}{\delta V_{BE}} |Q_{nB}| \\
|Q_{nB}| &= \frac{1}{2} q A_e w_B n_{pBO} e^{qV_{BE}/kT} \\
&= \frac{1}{2} w_B \left( \frac{w_B}{D_{nB}} \right) \left( \frac{qA_e D_{nB}}{w_B} \right) n_{pBO} e^{qV_{BE}/kT} = \left( \frac{w_B^2}{2D_{nB}} \right) i_c \\
C_b &= \frac{\delta}{\delta V_{BE}} \left( \frac{w_B^2}{2D_{nB}} \right) i_c = \left( \frac{w_B^2}{2D_n} \right) g_m \\
\frac{w_B^2}{2D_n} &= \tau_F \quad \text{base diffusion transit time}
\end{align*}
\]
$C_b$ is in between base and emitter:

$$C_b + C_{JE} = C_\pi$$

Add the following

- depletion capacitance: collector to bulk $C_{CS}$
- parasitic resistances: $\gamma_b$ of base, $\gamma_{ex}$ of emitter, $\gamma_c$ of collector

Complete small signal model