Recitation 20: Amplifiers Review

Yesterday, we introduced two more amplifier circuits: C-drain, C-base.
As we know, there is an analogy between MOS & BJT:

<table>
<thead>
<tr>
<th>MOS</th>
<th>BJT</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Source ↔</td>
<td>Common-Emitter</td>
<td>Voltage or $G_m$ Amp.</td>
</tr>
<tr>
<td>Common Drain ↔</td>
<td>Common-Collector</td>
<td>Voltage Buffer</td>
</tr>
<tr>
<td>Common Gate ↔</td>
<td>Common-Base</td>
<td>Current Buffer</td>
</tr>
</tbody>
</table>

Note: Buffer is an amplifier with gain 1, but input or output impedance changed.
We have also learned that there are 4 types of amplifiers, their two port models are:

![Diagrams of Amplifiers](image)

For the above single stage amplifiers (i.e. CS, CD, CG, CE, CC, CB), as we identify their particular function, e.g. current buffer is a type of current amplifier. We can use a two-port model for current amplifier to model a CB or CG circuit. Their corresponding $R_{in}$, $R_{out}$, $A_{io}$ will depend on the circuit (or device parameter), which we can derive based on the small signal circuit model of the circuit.

Yesterday, we looked at the example of CD & CG. Today we will look at CC & CB.
Common-Base Amplifier

Cast this into two port model

Need to find what is the corresponding $A_{io}, R_{in}, R_{out}$

$A_{io}$

Intrinsic current gain: ignore $R_s$, just consider $i_{in} = i_s; R_L$ short.

replace the

short $R_L$ at the output
\[ i_{\text{in}} = -\left( \frac{v_\pi}{\gamma_\pi} + g_m v_\pi + \frac{v_\pi}{\gamma_o} \right), \quad i_{\text{out}} = g_m v_\pi + \frac{v_\pi}{\gamma_o} \]

\[ \implies v_\pi = \frac{-i_{\text{in}}}{1/v_\pi + g_m + 1/\gamma_o} = \frac{i_{\text{in}}}{g_\pi + g_m + g_o} \]

\[ \implies A_{i_o} = \frac{i_{\text{out}}}{i_{\text{in}}} = -\frac{(g_m + g_o) \cdot \frac{i_{\text{in}}}{g_\pi + g_m + g_o}}{i_{\text{in}}} = -\frac{g_m + g_o}{g_\pi + g_m + g_o} \approx -1 \]

\[ \therefore \frac{1}{g_m} \approx 1 \text{k} \Omega, \quad \gamma_o \approx 100 \text{k} \Omega \]

\[ g_m \gg g_o, \quad \gamma_\pi = \frac{\beta_F}{g_m} \implies g_\pi = \frac{g_m}{\beta_F} \quad g_\pi \ll g_m \]

\[ R_{\text{in}} \]

---

replace \[ \begin{array}{c}
\begin{array}{c}
\omega_x \\
\times \omega_y
\end{array}
\end{array} \]

with \[ \begin{array}{c}
\begin{array}{c}
\omega_x \\
\times \omega_y
\end{array}
\end{array} \]

have \( R_L \) across output

\[ \gamma_\pi, \gamma_o \gg \frac{1}{g_m} \] as we just discussed

\[ \therefore \text{transconductance generator } g_m \text{ dominates currents at the input node} \]

\[ i_t = -\left( \frac{v_\pi}{\gamma_\pi} + g_m v_\pi + \frac{v_o}{\gamma_o} \right) \approx -g_m v_\pi = g_m v_t \]

\[ \therefore R_{\text{in}} = \frac{v_t}{i_t} = \frac{v_t}{g_m v_t} \approx \frac{1}{g_m} \quad \text{LOW! (good for getting current in)} \]

Exact: see pp 150 \( R_{\text{in}} = \frac{1}{\frac{1}{\gamma_\pi} + g_m + \frac{1 - g_m (\gamma_\text{co} | R_L)}{\gamma_o + (V_\text{oc} | R_L)}} \)
$R_{\text{out}}$

Similarly

1. shut down all independent sources
2. load input with $R_s$
3. put test current source at output
4. \[ R_{\text{out}} = \frac{v_t}{i_t} \]

\[
i_t = g_m v_t + \frac{v_t + v_\pi}{\gamma_o} \quad \text{voltage across } \gamma_o \text{ is } v_t + v_\pi \quad (1)
\]
\[
v_\pi = -i_t \cdot (\gamma_\pi || R_s) \quad (2)
\]

\[\Rightarrow\] plug (2) into (1)
\[
i_t = \frac{v_t / \gamma_o}{1 + \frac{\gamma_\pi || R_s}{\gamma_o} + g_m (\gamma_\pi || R_s)} \quad (3)
\]

\[\Rightarrow\]\[
\frac{v_t}{i_t} = \frac{\gamma_o + (\gamma_\pi || R_s) + g_m \gamma_o (\gamma_\pi || R_s)}{\gamma_o} \quad (4)
\]

\[\Rightarrow\]\[
\frac{v_t}{i_t} = \gamma_o || [\gamma_o + (\gamma_\pi || R_s) + g_m \gamma_o (\gamma_\pi || R_s)] \simeq \gamma_o || [1 + g_m (\gamma_\pi || R_s)]\beta_F \quad (5)
\]

If $R_s \gg \gamma_\pi$, \[ R_{\text{out}} \simeq \gamma_o || [1 + g_m \gamma_\pi] = \gamma_o || \gamma_o \cdot \beta_F \left(\frac{1}{\beta_p}\right) \quad (7)\]

Excellent current buffer: can use current source with source resistance only slightly higher than $R_{\text{in}} \left(\frac{1}{g_m}\right)$, and get same current with high $R_{\text{out}}$. 

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For the circuit shown, the voltage divider in the feedback path is $v_\pi$ across $\gamma_\pi || R_s$. The current source $i_t$ is amplified by $\gamma_o$, so the output current $i_t$ is given by $i_t = \gamma_o v_t$. The voltage across $\gamma_o$ is $v_t + v_\pi$, and the voltage across $\gamma_\pi || R_s$ is $v_\pi$. The current through $R_s$ is $i_t \cdot (\gamma_\pi || R_s)$.
Common-Collector Amplifier

Rearrange,
Cast this into two port voltage amplifier model

\[ A_{vo}(R_L = \infty, R_s = 0) \]

\[
\begin{align*}
V_{out} &= A_{vo} V_{in} = \left( g_m v_\pi + g_m \frac{v_\pi}{\beta_F} \right) \cdot (\gamma_o || \gamma_{oc}) \\
&= g_m \left( 1 + \frac{1}{\beta_F} \right) v_\pi (\gamma_o || \gamma_{oc}) \\
i_{in} &= \frac{v_\pi}{\gamma_\pi} = v_\pi \frac{g_m}{\beta_F} \\
\text{But } v_\pi &= v_{in} - v_{out} \implies v_{out} = g_m \left( 1 + \frac{1}{\beta_F} \right) (v_{in} - v_{out})(\gamma_o || \gamma_{oc}) \\
A_{vo} &= \frac{v_{out}}{v_{in}} = \frac{1}{1 + \frac{1}{g_m} \left( 1 + \frac{1}{\beta_F} \right) (\gamma_o || \gamma_{oc})} \approx 1
\end{align*}
\]

\[ R_{in} \]

Leave \( R_L \) in place, replace source with

\[
\begin{align*}
v_t &= i_t \cdot \gamma_\pi + (i_t + g_m v_\pi) \cdot (\gamma_o || \gamma_{oc} || R_L) \\
&= i_t v_\pi + g_m \left( 1 + \frac{1}{\beta_F} \right) v_\pi (\gamma_o || \gamma_{oc} || R_L) \\
R_{in} &= \frac{v_t}{i_t} = \gamma_\pi + \frac{g_m \left( 1 + \frac{1}{\beta_F} \right) v_\pi (\gamma_o || \gamma_{oc} || R_L)}{g_m \beta_F} \\
&= \gamma_\pi + (\beta_F + 1)(\gamma_o || \gamma_{oc} || R_L) \quad \text{much larger than } \gamma_\pi
\end{align*}
\]
$R_{\text{out}}$

$v_s = 0$, leave $R_s$, apply $v_t, i_t$ at the output

\[ (i_t + g_m v_\pi + \frac{v_t}{\beta_F}) \cdot (\gamma_o || \gamma_{oc}) = v_t \]

\[
\text{voltage divider } v_\pi = -\frac{\gamma_0}{\gamma_0 + R_s} \cdot v_t
\]

\[
\implies i_t = -g_m v_\pi + \frac{v_t}{\gamma_0 || \gamma_{oc}}
\]

\[
\implies i_t = \frac{g_m \gamma_0}{\gamma_0 + R_s} \cdot v_t + \frac{v_t}{\gamma_0 || \gamma_{oc}} = \left( \frac{\beta_F}{\gamma_0 + R_s} + \frac{1}{\gamma_0 || \gamma_{oc}} \right) v_t
\]

\[
\therefore i_t \approx \frac{\beta_F}{\gamma_0 + R_s} v_t
\]

\[
R_{\text{out}} = \frac{v_t}{i_t} = \frac{\gamma_0 + R_s}{\beta_F} = \frac{1}{g_m} + \frac{R_s}{\beta_F} \quad \text{LOW!} \quad \therefore g_m, \beta_F \text{ large}
\]

In conclusion, see the summary sheet handout