Recitation 21: Intrinsic Frequency Response of CS & CE Amplifier

Yesterday, we discussed the intrinsic frequency response of the CE Amplifier. Since there is an analogy between MOSFET & BJT, today we will look at the intrinsic frequency response of a CS Amplifier and compare them.

Small Signal Model

<table>
<thead>
<tr>
<th>BJT</th>
<th>MOSFET</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_\pi = C_{je} + C_b = C_{je} + g_m \tau_F = C_{je} + g_m \left( \frac{wD}{2D_n} \right)$</td>
<td>$C_{gs} = \frac{2}{3} wLC_{ox} + wC_{ov}$</td>
</tr>
<tr>
<td>$C_\mu$: depletion capacitance only</td>
<td>$C_{gd} = wC_{ov}$</td>
</tr>
</tbody>
</table>

$V_{sb} = 0$: so only 3 terminals
**Intrinsic Frequency Response:** $R_s \to \infty \quad R_L = 0$

### BJT

Node 1: $i_{in} = \frac{v_{\pi}}{\gamma_{\pi}} + jwC_{\pi} \cdot v_{\pi} + jwC_{\mu}v_{\pi}$

Node 2: $i_{out} = g_m v_{\pi} - jwC_{\mu}v_{\pi}$

$$\frac{i_{out}}{i_{in}} = \frac{g_m \gamma_{\pi} \left(1 - \frac{jwC_{\mu}}{g_m}\right)}{1 + jw\gamma_{\pi}(C_{\pi} + C_{\mu})} \beta_{o} \left(1 - \frac{jwC_{\mu}}{g_m}\right)$$

$$= \frac{1 + jw\gamma_{\pi}(C_{\pi} + C_{\mu})}{1 + jw\gamma_{\pi}(C_{\pi} + C_{\mu})}$$

### MOSFET

Node 1: $i_{in} = jwC_{gs} \cdot V_{gs} + jwC_{gd}V_{gs}$

Node 2: $i_{out} = g_m v_{\pi} - jw$

$$\frac{i_{out}}{i_{in}} = \frac{g_m - jwC_{gd}}{jw(C_{gs} + C_{gd})}$$

---

**Logarithmic plots:**

- **BJT:** $\log\left|\frac{i_{out}}{i_{in}}\right|$ vs. $\log\omega$
- **MOSFET:** $\log\left|\frac{i_{out}}{i_{in}}\right|$ vs. $\log\omega$
Unit Gain Frequency, $f_T$

<table>
<thead>
<tr>
<th>BJT</th>
<th>MOSFET</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_T = \frac{1}{2\pi} \cdot \frac{w_T}{g_m} = \frac{1}{2\pi} \frac{g_m}{C\mu + C\pi}$</td>
<td>$f_T = \frac{1}{2\pi} \frac{g_m}{C_{gs} + C_{gd}}$</td>
</tr>
</tbody>
</table>

Frequency at which the current gain is reduced to 1(0 dB)

This is obtained by:

$$\left| \frac{i_{out}}{i_{in}} \right| = \left| \frac{\beta_o \left(1 - \frac{jwC\mu}{g_m}\right)}{1 + jw\gamma(C\mu + C\pi)} \right| = 1$$

Ignoring the zero on top, $\therefore \frac{g_m}{C\mu} \gg w_T$

$$\left| \frac{1 + jw\gamma(C\mu + C\pi)}{\frac{1}{\gamma(C\mu + C\pi)}} \right| = 1 \quad \therefore w_T \gg \frac{1}{\gamma(C\mu + C\pi)} \quad w_T \gamma(C\mu + C\pi) \gg 1$$

$$\left| \frac{\beta_o}{jw\gamma(C\mu + C\pi)} \right| = 1 \quad \Rightarrow \quad w_T = \frac{g_m}{C\mu + C\pi}$$

$$f_T = \frac{1}{2\pi} \frac{I_c/V_{th}}{\tau_F} + C_j + C\mu \quad (\therefore g_m = \frac{I_c}{V_{th}})$$

$\frac{\mu_n V_{DSAT}}{L} \sim \text{velocity of carrier}$

$$\frac{\mu_n V_{DSAT}}{L} = 1/\tau_T = \tau_T = L/\text{velocity}$$

$\tau_T$ is transit time from source to drain

$f_T$ is independent of $V$.

For high frequency performance, NMOS $>$ PMOS.

Scale $L$ as short as possible.
At low $I_c$, $f_T$ is dominated by depletion capacitances at Base-emitter and base collector junctions ($C_{je}$ and $C_{\mu}$). As $I_c \uparrow$, diffusion capacitance $g_m \tau_F \uparrow$, and becomes dominant.

Fundamental limit for frequency response

$$\tau_F = \frac{w_B^2}{2D_{n,p}}$$

To increase $f_T$
- high $I_c = $ diffusion cap. limited $\implies$ shrink base width.
- low $I_c = $ depletion cap. limited $\implies$ shrink device area

Another note for MOSFET: the current gain $\to \infty$ at $w = 0$. This is because of gate oxide, DC input current = 0. MOSFET not used as current amplifier at low frequency (input resistance too high)