Problem 12.1

A certain clad optical fiber can be approximated by a dielectric slab waveguide of thickness $D_1$ and permittivity $\varepsilon_1 = 4$ that is centered in a dielectric of thickness $D_2$ and permittivity $\varepsilon_2 = 3.9$, as illustrated. Assume the angle of incidence within the inner fiber is $\theta_i = 85^\circ$, which is beyond the critical angle $\theta_c$.

(a) What is the critical angle $\theta_c$?

(b) In terms of the free-space wavelength $\lambda_o$, what is the 1/e decay length $\alpha^{-1}$ in the cladding ($\varepsilon_2$), where $\vec{E}(x) = \hat{y} E_0 e^{-ax}$? Normally $D_2$ is made many times thicker than $\alpha^{-1}$ to ensure no external attenuation occurs.

(c) Roughly guess what $D_1$ is in terms of $\lambda_o$ for the TE$_1$ mode. Briefly explain your reasoning.

Problem 12.2

A four-level laser has equally spaced atomic energy levels $E_4 > E_3 > E_2 > E_1$, as illustrated. Assume the spontaneous transition rates $A_{41}$, $A_{42}$, $A_{31}$, and $A_{32} \approx 0$, while the others are larger.

(a) When the 1-4 transition is pumped, in which energy level do most atoms eventually find themselves in the absence of lasing? Briefly explain your reasoning.

(b) For this case, approximately what is the maximum possible energy efficiency of this laser ($P_{out}/P_{pump}$) for the 3-2 laser transition?
Problem 12.3

A laser diode has $\varepsilon = 4\varepsilon_o$ and a cavity length 1-mm long terminated by one perfect mirror and one mirror with a power transmission coefficient of 1 percent. It operates at 1-micron wavelength and with a natural linewidth of 0.1 percent associated with the width of the energy levels involved ($Q_{\text{natural linewidth}} \approx 10^3$).

(a) What is the frequency spacing (Hz) between resonances for this laser cavity?

(b) Approximately how many different frequency lines are emitted by this laser within its overall natural bandwidth?

(c) What is the approximate width $\Delta f$ of each of these lines (Hz) associated with the cavity $Q_c$? A numerical answer is desired, so $Q_c$ must be estimated.

Problem 12.4

A rectangular acoustic waveguide measures 5×5 millimeters, similar to the ear canal that runs between the outer ear and the eardrum.

(a) What are the cutoff frequencies $f_{0,3}$ for the lowest four acoustic modes for this waveguide? Assume $c_s = 340$ ms$^{-1}$.

(b) What are the two lowest resonant frequencies $f_{001}$ and $f_{002}$ for the resonator bounded by the eardrum and outer ear for an ear canal that is 2-cm long.

(c) Discuss briefly (guess if necessary) how and why the frequencies found in parts (a) and (b) may relate to the nominal cutoff for human hearing, which is typically 12-20 kHz.

(d) The outer human ear has a ridge that helps focus on the ear canal sounds arriving from the direction in which that person is facing. If a single ridge is approximately three centimeters from the ear canal, as illustrated, what frequencies are favored for this forward focusing effect? What audible frequencies are partially nulled? Note that there are two ridges in most ears, and their spacing is a function of direction. Explain briefly how this could help our cavemen ancestors determine the direction of an unseen predator.

(e) There is a substantial acoustic impedance discontinuity at the junction of the ear canal and the outer ear characterized by the power reflection coefficient $|\Gamma|^2$ at the junction. What is the approximate external $Q_E$ of the lowest frequency mode for this ear-canal acoustic resonator in terms of $|\Gamma|^2$? Note that the power lost to the resonator (ear canal) via radiation corresponds to power dissipated in the expression for $Q$, where $Q_E = Q_L$ here.
**Problem 12.5**

The velocity of sound in the atmosphere is a function of temperature, and has a nominal value of \( c_s \approx 340 \text{ m s}^{-1} \). Although air temperatures typically drop with altitude \(-7 \text{ K km}^{-1}\), there can be temperature inversions for which the air temperature actually increases with altitude, such as when a warm air mass moves in over a cold surface like that of a lake. The acoustic impedance for this problem is \( \eta_s \approx 425 \).

(a) If 100 watts of 1-kHz acoustic power were confined as a plane wave within 1 square meter, what would be the peak acoustic pressure \( |p| \) associated with it?

(b) What would be the peak acoustic velocity \( |u| \)?

(c) What is the peak-to-peak motion \( D \) [m] of the air molecules in this wave?

(d) If this wave were travelling upward and were incident on a warmer air mass with \( c_{s,\text{warm}} \) one percent greater than \( c_s \) for the cooler air mass below, what would be the critical angle \( \theta_c \) for this acoustic wave?

(e) If the angle of incidence on this warmer air mass equals \( \theta_c \) so that an evanescent acoustic wave is produced above the interface that decays as \( e^{-\alpha z} \), where \( z \) is altitude, what is the value of \( \alpha \) in this singular case?

**Problem 12.6**

A typical closed door consists of a thin massive planar sheet blocking the acoustic path. Assume its density \( \rho_d = 1000 \text{ kg/m}^3 \) while that of air is \( \rho_o \approx 1 \text{ kg/m}^3 \). The corresponding velocities of sound \( c_s \) are 1050 and 330 m/s, respectively. Assume normal incidence for all acoustic waves.

(a) What is the ratio of the characteristic acoustic impedances \( \eta_o/\eta_d \) for the air and the door? \( (\eta = \rho c_s \) is analogous to \( Z_o \).)

(b) If the door were infinitely thick, what fraction of the incident acoustic power would the door reflect?

(c) What is the lowest acoustic frequency \( f_{\text{pass}} \) (if any) that could pass through this ideal lossless 5-cm thick door without any attenuation whatever?

(d) This 5-cm thick door has maximum reflectivity \( |\Gamma|^2 \) at certain frequencies, i.e. its maximum power to stop sound; what is the lowest such frequency \( f_{\text{stop}} \) and what is the corresponding \( |\Gamma|^2 \)?