Student Name:

Final Exam  Closed book, no calculators

Please note the two pages of formulas provided at the back; the laser and acoustic expressions have been revised slightly. There are 10 problems; some are on the back sides of the sheets. For full credit, please simplify all expressions, present numerical answers to the extent practical without a calculator or tedious computation, and place your final answers within the boxes provided. You may leave natural constants and trigonometric functions in symbolic form (π, ε₀, μ₀, η₀, h, e, sin(0.9), √2, etc.). To receive partial credit, provide all related work on the same sheet of paper and give brief explanations of your answer. Spare sheets are at the back.

Problem 1. (25/200 points)

Two square capacitor plates in air have separation d, sides of length b, and charge ±Q as illustrated. Fringing fields can be neglected.

a) What is the capacitance \( C_a \) of this device?

\[
C_a = \frac{\varepsilon_0 b^2}{d}
\]
b) A perfectly conducting plate is introduced between the capacitor plates, leaving parallel gaps of width $d/10$ above and below itself. What now is the device capacitance $C_b$ when it is fully inserted?

$$C_b = \frac{\varepsilon_0}{d/10}$$

c) What is the magnitude and direction of the force $\vec{f}$ on the new plate of Part (b) as a function of the insertion distance $L$. Please express your answer as a function of the parameters given in the figure.

$$\vec{f} = \frac{2Q}{\varepsilon_0 L}$$
**Problem 2.** (20/200 points)

The plate separation of a lossless parallel-plate TEM line many wavelengths long (length $D = 100.25\lambda$) very slowly increases from end A to end B, as illustrated. This increases the characteristic impedance of the line from $Z_o$ at the input end A, to $4Z_o$ at the output end B. This transition from A to B is so gradual that it produces no reflections. End B is terminated with a resistor of value $4Z_o$.

![Diagram of a lossless parallel-plate TEM line with increasing plate separation from A to B, and a resistor at B.]

a) What is the input impedance $Z_A$ seen at end A? Explain briefly.

$$Z_A = \ldots$$

Explanation:

b) If the sinusoidal (complex) input voltage is $V_A$, what is the output voltage $V_B$?

$$V_B = \ldots$$

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*Please turn sheet over for Problem 3.*
**Problem 3.** (25/200 points)

At \( t = 0 \) a switch connects a voltage \( V \) to a passive air-filled short-circuited TEM line of length \( D \) and characteristic impedance \( Z_0 \), as illustrated. Please sketch and quantify dimension:

a) The line voltage \( v(z) \) at \( t = D/2c \).

b) The current \( i_B(t) \) through the short circuit for \( 0 < t < 2D/c \).

c) The current \( i_A(t) \) from the voltage source \( (z = 0) \) for \( 0 < t < 3D/c \).
Problem 4. (30/200 points)

A 100-ohm air-filled lossless TEM line is terminated with a 100-ohm resistor and a $10^{-10}/2\pi$ Farad capacitor in series, as illustrated. It is driven at 100 MHz.

a) What fraction $F = |\Gamma_L|^2$ of the incident power is reflected from this load?

$$F = \ldots$$

b) What is the minimum distance $D$ (meters) from the load at which the line current $|I(z)|$ is maximum? You may express your answer in terms of the angle $\beta$ (degrees) shown on the Smith Chart.

$$D_{\text{min}} = \ldots$$

Please turn sheet over to answer part (c).
c) Can we match this load by adding another capacitor in series somewhere and, if so, at what distance D and with what value $C_m$?

<table>
<thead>
<tr>
<th>Can we match?</th>
<th>YES</th>
<th>NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>D =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_m$ =</td>
<td></td>
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</tr>
</tbody>
</table>
**Problem 5.** (20/200 points)

A flat perfect conductor has a surface current in the xy plane at $z = 0$ of:

$$
\bar{J}_x = J_0 e^{-jbx} \text{ [A/m]}
$$

a) Approximately what is $\vec{H}$ in the xy plane at $z = 0^+$?

$$
\vec{H}(z = 0^+) =
$$

b) How might one easily induce this current sheet at frequency $f$ [Hz] on the surface of a good conductor? Please be reasonably specific and quantitative.

To induce this current one might:

Please turn sheet over for Problem 6.
Problem 6. (10/200 points)

A certain evanescent wave at angular frequency $\omega$ in a slightly lossy medium has $\vec{E} = \gamma E_0 e^{\alpha(x-0.01z)} - jbz$, assume $\mu = \mu_0$. What is the distance $D$ between phase fronts for this wave?

$$D = \Box$$
Problem 7. (25/200 points)

A resonator is filled with a dielectric having \( \varepsilon = 4\varepsilon_0 \) and has dimensions \( b, a, \) and \( d \) along the \( x, y, \) and \( z \) directions, respectively, where \( d > a > b \).

a) What is the lowest resonant frequency \( f_{m,n,q} \) [Hz] for this resonator?

\[
f_{m,n,q} \text{ [Hz]} =
\]

b) What is the polarization of the electric vector \( \vec{E} \) at the center of the resonator for this lowest frequency mode?

Polarization of \( \vec{E} \) is:

Please turn sheet over to answer part (c).
c) What is the Q of this resonance if the dielectric has a slight conductivity $\sigma$? Hint: a ratio of integrals may suffice, so the integrals might not need to be computed.

$$Q = \boxed{\text{ }}$$
Problem 8. (20/200 points)

A certain transmitter transmits $P_T$ watts of circularly polarized radiation with antenna gain $G_o$ (in circular polarization) toward an optimally oriented matched short-dipole receiving antenna (gain $= 1.5$) located a distance $R$ away. The wavelength is $\lambda$.

\[ P_T [W] \]

\[ \text{at } \lambda \]

\[ G_o \]

\[ \text{Circular polarization} \]

\[ R/2 \]

\[ R \]

\[ \text{Obstacle} \]

\[ P_{\text{rec}} [W] \]

a) In the absence of any obstacles or reflections, what power $P_R$ is received?

\[ P_R = \]

Please turn sheet over to answer part (b).
b) A large metal fence is then erected half way between the transmitter and receiver, and perpendicular to the line of sight. Fortunately it has a round hole of area $A$ centered on that line of sight. Assume the hole is sufficiently small that the electrical phase of the incident wave is constant over its entirety. What power is received now?

$$P_R =$$
**Problem 9.** (15/200 points)

An ideal lossless three-level laser has the illustrated energy level structure. Level 1 is $\Delta$ Joules above the ground state, and Level 2 is $3\Delta$ Joules above the ground state. All rates of spontaneous emission $A_{ij}$ have the same finite value except for $A_{21}$, which is infinite.

a) What should be the laser frequency $f_L$ [Hz]?

$$f_L \text{ [Hz]} =$$

b) What is this laser’s maximum possible efficiency $\eta = (\text{laser power})/(\text{pump power})$?

$$\eta =$$

*Please turn sheet over for Problem 10.*
Problem 10. (10/200 points)

Two monopole (isotropic) acoustic antennas lying on the z axis are aligned in the z direction and separated by 2\(\lambda\), as illustrated. They are fed 180° out of phase. In what directions \(\theta\) does this acoustic array have maximum gain \(G(\theta)\)? Simple expressions suffice. If more than one direction has the same maximum gain, please describe all such directions.

\[
\theta = \text{\ldots}
\]