Problem 1. (34/100 points)

A uniform plane wave in an unusual gaseous medium characterized by $\mu, \varepsilon$ is represented by:

$$\mathbf{E} = 2 \hat{x} \cos(t + z) \quad \text{and} \quad \mathbf{H} = \hat{y} \sin(t + z + \frac{\pi}{2}).$$

In each case below please briefly indicate your method, equations, or reasoning. What are the numerical values for:

a) The wave velocity $v$ and direction [m/s]?

b) The time average wave intensity $I$ [W/m$^2$]? 

c) The permeability $\mu$ [H/m] for this medium?

d) The pressure $P_m$ [N/m$^2$] the wave exerts when normally incident ($\theta_i = 0$) on a perfectly reflecting mirror?

Problem 2. (18/100 points)

A factory measures the velocity $v_m$ [m/s] of molten metal running down an insulating sluice of width $W$ by measuring the voltage $V$ across the width of the channel produced by the illustrated uniform vertical magnetic field $B$. What is the velocity $\mathbf{v}_m$ in terms of the other given parameters? Briefly explain your reasoning, including the direction of $\mathbf{v}_m$ (see illustration).
Problem 3. (20/100 points)

What constraints are imposed on $\overrightarrow{H}_0$ in free space at the flat surface of a medium having $\sigma = 0$ and $\mu = \infty$? Briefly explain your reasoning.

Problem 4. (28/100 points)

The illustrated inductor consists of $N$ turns of wire uniformly wound around a thin hollow toroid with a major diameter of $D$ to produce an inductance of $L$ [Henries]. The toroid cross-sectional area is $A$, as illustrated, and you may neglect any fields outside the toroid.

a) What is $H$ inside the toroid when the current through the coil is $I$ Amperes?

b) What is the approximate inductance $L$ of this inductor?

c) The core of this coil is now one-percent filled with many turns of iron wire having $\mu = 1000\mu_0$, as illustrated. What now is the inductance $L$?