### 6.013 Electromagnetics and Applications – Quiz 2 Solutions

#### Problem 1. (28/100 points)
a) \( R_{Th} = 100 \, \text{ohms} \) since there is no "glitch" at \( t = 2\tau \) and therefore no reflection at the source when the step returns at \( 2\tau \). Thus \( R_{Th} \) is matched to the line.

b) Line length \( D = c\tau/2 \)

c) An \( R \) and \( C \) in parallel at the end of the line produces this response; the \( C \) in parallel looks like a short circuit, yielding the zero at \( t = \tau \), and the \( R \) in parallel prevents the voltage \( V \) from returning to the source voltage \( A = 2 \, \text{volts} \).

#### Problem 2. (22/100 points)
A certain parallel-plate TEM transmission line is filled with \( \mu_0 \) and \( \varepsilon = 9\varepsilon_0 \).

a) A 1-GHz signal on this TEM line has \( \lambda = 10 \, \text{cm} \) since \( \lambda = v_p/f = 1/10^{9}\sqrt{\mu_0 9\varepsilon_0} = 0.1[\text{m}] \).

b) \( |V(z)| = V_0 e^{-az} \). \( V_0 e^{-j\omega \sqrt{\mu_0 9\varepsilon_0} (1+0.01)j}z = V_0 e^{-j\omega \sqrt{1+0.01}j}z \), \( \sqrt{1+0.01} = \pm(1+0.005) \), but \( - \) solution propagates in the wrong direction. Therefore we have \( e^{-j\omega (1+0.005)j}z \rightarrow e^{0.1\pi} = e^{-az} \) Therefore this transmission line amplifies.

#### Problem 3. (22/100 points)
a) \( \vec{H} = \hat{x} H_0 e^{jz \cdot 0.6y} \) is a TEM wave because \( H \) is \( \perp \) \( z \) axis, the axis of propagation.

b) \( (\omega [\text{r/s}] / c)^2 = k_x^2 + k_y^2 = 1^2 + (0.6)j = 0.64 \). So \( \omega = 0.8c = 2.4 \times 10^8 \, [\text{r/s}] \).

#### Problem 4. (28/100 points)
(a) Numerical value of the inductance \( L \) [\( \text{Hy/m} \)] on a line having \( c = 3 \times 10^8 \, [\text{m/s}] \) and \( Z_o = 100 \) is \( L = \sqrt{\frac{1}{V_C}} \sqrt{LC} = 100/(3 \times 10^8) = 3.33 \times 10^{-7} \, [\text{Hy/m}] \)

(b) \( |\Gamma| = (\text{VSWR} - 1)/(\text{VSWR} + 1) = 2/4, \) so \( |\Gamma|^2 = 1/4 \).

(c) \( Z(z = -3 \, \text{meters}) = \) real and minimum since \( |V| = \) minimum. \( Z_o \) \( (z = -3) = (1 - |\Gamma|)/(1 + |\Gamma|) = 1/3, \) so \( Z \) \( (z = -3) = 100/3 = 33.3 \, \Omega \).