MEDIA AND BOUNDARY CONDITIONS

Media: conductivity $\sigma$, permittivity $\varepsilon$, permeability $\mu$

Media are the only tools we have to create or sense EM fields

Conductivity ($\sigma$):

$\bar{J}$ (current density, A/m$^2$) = $nq\langle \vec{v} \rangle = \sigma \vec{E}$$
n = \#q's/m^3$, $q =$ charge (Coulombs), $\langle \vec{v} \rangle =$ average velocity (m/s)

Semiconductors:

$P\{\text{escape}\} \propto e^{-W/kT}$

Metals:

$J \propto \langle v \rangle = \langle at \rangle = \left\langle \frac{f}{m} t \right\rangle = \frac{qE}{m} \langle t \rangle$, $\therefore \sigma \propto \frac{q}{m} \langle t \rangle$; $\langle t \rangle = f(T_{emp})$

($t =$ time before collisions reset $v \approx 0$)
DIELECTRICS

Vacuum:
\[ \vec{D} = \varepsilon_0 \vec{E} \quad \iint_S \vec{D} \cdot \hat{n} \, da = \iiint_V \rho_f \, dv \]
\( \rho_f \) = free charge density

Dielectric Materials:
\[ \vec{D} = \varepsilon \vec{E} = \varepsilon_0 \vec{E} + \vec{P} \]
\[ \iint_S \varepsilon_0 \vec{E} \cdot \hat{n} \, da = \iiint_V (\rho_f + \rho_p) \, dv \]
\[ \iint_S \vec{P} \cdot \hat{n} \, da = -\iiint_V \rho_p \, dv \]
\( \rho_p \) is polarization (surface) charge density
\( \vec{P} = \) “Polarization Vector”
MAGNETIC MATERIALS

Basic Equations:
\[ \iiint_S \mathbf{B} \cdot \hat{n} \, da = 0 \]
\[ \mathbf{B} = \mu_0 \mathbf{H} \text{ in vacuum} \]
\[ \mathbf{B} = \mu \mathbf{H} = \mu_0 \left( \mathbf{H} + \mathbf{M} \right) \]
\[ \mathbf{M} = \text{“Magnetization Vector”} \]
\[ \mu = \text{permeability} \]

- \( \mathbf{B} \): Magnetic field
- \( \mathbf{H} \): Magnetic field intensity
- \( \mathbf{M} \): Magnetization
- \( \mu \): Permeability
- \( \mu_0 \): Free space permeability
- \( \hat{n} \): Unit normal vector
- \( da \): Surface element

Pictorial representations illustrate the concepts of magnetic domains and domain walls.
SATURATION AND HYSTERESIS

\[ \frac{1}{2} \mathbf{B} \cdot \mathbf{H} = W_m \] [J/m\(^3\)] Magnetic energy density

- H \(_c\) magnetic coercive force
- Residual flux density B\(_r\)
- Area = hysteresis loss [J/m\(^3\)]
# MEDIA PARAMETERS

<table>
<thead>
<tr>
<th>Conductivity $\sigma$ [Siemens/m]</th>
<th>Dielectric constant ($\varepsilon/\varepsilon_0$)</th>
<th>Relative permeabilities $\mu/\mu_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paraffin</td>
<td>Vacuum</td>
<td>Bismuth</td>
</tr>
<tr>
<td>Glass</td>
<td>Wood (fir)</td>
<td>Silver</td>
</tr>
<tr>
<td>Dry earth</td>
<td>Teflon, petroleum</td>
<td>Copper</td>
</tr>
<tr>
<td>Distilled water</td>
<td>Vaseline</td>
<td>Water</td>
</tr>
<tr>
<td>Sea water</td>
<td>Paper</td>
<td>Vacuum</td>
</tr>
<tr>
<td>Iron</td>
<td>Polystyrene</td>
<td>Air</td>
</tr>
<tr>
<td>Copper</td>
<td>Sandy soil</td>
<td>Aluminum</td>
</tr>
<tr>
<td>Silver</td>
<td>Fused quartz</td>
<td>Cobalt</td>
</tr>
<tr>
<td></td>
<td>Ice</td>
<td>Nickel</td>
</tr>
<tr>
<td></td>
<td>Pyrex glass</td>
<td>Mild steel</td>
</tr>
<tr>
<td></td>
<td>Aluminum oxide</td>
<td>Iron</td>
</tr>
<tr>
<td></td>
<td>Ethyl alcohol</td>
<td>Mu metal</td>
</tr>
<tr>
<td></td>
<td>Water</td>
<td>Supermalloy</td>
</tr>
<tr>
<td></td>
<td>Titanium dioxide</td>
<td></td>
</tr>
</tbody>
</table>

- Paraffin: $\sim 10^{-15}$
- Glass: $10^{-12}$
- Dry earth: $10^{-4}$-$10^{-5}$
- Distilled water: $2 \times 10^{-4}$
- Sea water: 3-5
- Iron: $10^7$
- Copper: $5.8 \times 10^7$
- Silver: $6.1 \times 10^7$
INTEGRAL MAXWELL’S EQUATIONS

Graphical Equations:

\[ \iiint_S \mathbf{D} \cdot \hat{n} \, da = \iiint_V \rho \, dv \]

\[ \iiint_S \mathbf{B} \cdot \hat{n} \, da = 0 \]

\[ \oint_c \mathbf{E} \cdot ds = -\frac{\partial}{\partial t} \iint_A \mathbf{B} \cdot \hat{n} \, da \]

\[ \oint_c \mathbf{H} \cdot ds = \iint_A \mathbf{J} \cdot \hat{n} \, da + \frac{\partial}{\partial t} \iint_A \mathbf{D} \cdot \hat{n} \, da \]

\[ \mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H} \]

Constitutive relations
FIELDS PERPENDICULAR TO BOUNDARIES

Using Gauss’s Law: \( \iiint_S \vec{D} \cdot \hat{n} \, da = \iiint_V \rho \, dv \):

\[ \iint_S \vec{D} \cdot \hat{n} \, da \rightarrow (D_{1\perp} - D_{2\perp}) A = \rho_s A \]

(Lim \( A \rightarrow 0, \delta^2 << A \))

Therefore:

\[ D_{1\perp} - D_{2\perp} = \rho_s \] yields:

\[ \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s \]

\[ \iint_A \vec{B} \cdot \hat{n} \, da = (B_{1\perp} - B_{2\perp}) A = 0 \] yields:

\[ \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0 \]
FIELDS PARALLEL TO BOUNDARIES

Using Faraday’s Law:

\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \iint_A \vec{B} \cdot \hat{n} d\vec{a} \]

\[ \oint \vec{E} \cdot d\vec{s} \rightarrow (E_{1//} - E_{2//})L = -\frac{d}{dt} \iint_A \vec{B} \cdot d\vec{a} \rightarrow 0 \quad (\text{Lim} \ \delta \rightarrow 0) \]

Therefore:

\[ \begin{align*}
E_{1//} &= E_{2//} \\
\hat{n} \times (\vec{E}_1 - \vec{E}_2) &= 0
\end{align*} \]

Using Ampere’s Law:

\[ \oint \vec{H} \cdot d\vec{s} = \iint_A (\vec{J} + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{a} \]

\[ \oint \vec{H} \cdot d\vec{s} \rightarrow (H_{1//} - H_{2//})L = \iint_A \vec{J} \cdot d\vec{a} - \frac{\partial}{\partial t} \iint_A \vec{D} \cdot d\vec{a} \rightarrow 0 \]

Therefore:

\[ \hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \]
PERFECT CONDUCTORS

**Electric Fields:**

\[
\{ \text{if } \sigma \to \infty \text{ and } \mathbf{E} \neq 0 \} \Rightarrow \{ \mathbf{J} = \sigma \mathbf{E} \to \infty \} \Rightarrow \{ \mathbf{H} \to \infty \} \text{ since }
\]

\[
\oint_{C} \mathbf{H} \cdot d\mathbf{s} = \iint_{A} (\mathbf{J} + \partial \mathbf{D}/\partial t) \cdot d\mathbf{a} \Rightarrow \{ W_m = \mu H^2/2 \text{ [J/m}^3] \to \infty, \text{ and } w_m \to \infty \}
\]

Therefore:

\[
\begin{aligned}
\mathbf{E} &= 0 \text{ inside perfect conductors } \\
\rho &= 0 \text{ (since } \int_{V} \rho dv = \iint_{S} \varepsilon \mathbf{E} \cdot \mathbf{n} \, da \rangle
\end{aligned}
\]

**Magnetic Fields:**

\[
\{ \text{If } \mathbf{E} = 0 \text{ and } \frac{\partial}{\partial t} \iint_{A} \mathbf{B} \cdot \mathbf{n} da = -\oint_{C} \mathbf{E} \cdot d\mathbf{s} \} \Rightarrow \{ \partial \mathbf{B}/\partial t = 0 \}
\]

Therefore:

\[
\begin{aligned}
\mathbf{H} &= 0 \text{ inside perfect conductors } \\
&= \text{(if } \sigma = \infty, \text{ and } \mathbf{H}(t=0) = 0 \text{)}
\end{aligned}
\]

**Superconductors (Cooper pairs don’t impact lattice):**

\[
\mathbf{B} \cong 0 \text{ inside because } \sigma = \infty
\]

Cooper pairs of electrons disassociate and superconductivity fails when the external \( B(T) \) is above a critical threshold
SUMMARY: BOUNDARY CONDITIONS

General Boundary Conditions:
\[ \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s \]
\[ \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0 \]
\[ \hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0 \]
\[ \hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \]

Inside Perfect Conductors:
\[ \vec{D}_2 = \vec{B}_2 = \vec{E}_2 = 0 \]
\[ \hat{n} \cdot \vec{D}_1 = \rho_s \]
\[ \hat{n} \cdot \vec{B}_1 = 0 \]
\[ \hat{n} \times \vec{E}_1 = 0 \]
\[ \hat{n} \times \vec{H}_1 = \vec{J}_s \]

⇒ \( \vec{B} \) is parallel to perfect conductors
⇒ \( \vec{E} \) is perpendicular to perfect conductors