Electromagnetics and Applications
• Maxwell’s equations: statics, quasistatics, and wave phenomena
• Applications: wireless, media, circuits, forces and generators, computer speed, microwaves, antennas, photonics, acoustics, etc.

Mathematical Methods
• Partial differential and difference equations, phasors, vector calculus

Problem Solving Techniques
• Perturbation, boundary-value, and energy methods; duality

Academic Review
• Mechanics, quantum phenomena, devices, circuits, signals, linear systems

Capstone Subject—Professional Preparation

Follow-on Subjects:
Electromagnetic waves: 6.632, Quasistatics: 6.641
ACOUSTIC ANTENNAS

Monopole Radiators:

\[ \nabla p = -\rho_o \frac{\partial \bar{u}}{\partial t} \quad [\text{Nm}^{-3}] \quad [\text{kg m}^{-2}\text{s}^{-2}] \]

Wave equation: \( (\nabla^2 + k^2)p = 0^* \quad (k = \omega/c_s) \)

\( \partial/\partial \theta = \partial/\partial \phi = 0 \quad \text{(radial source)} \)

Yields: \( \frac{d^2p}{dr^2} + \left(\frac{2}{r}\right)\frac{dp}{dr} + k^2p = 0 \)

Equivalent to: \( \frac{d^2(rp)}{dr^2} + k^2(rp) = 0 \)

General solution: \( rp \propto e^{\pm jkr} \)

Radiation outward: \( p(r) = (A/r)e^{-jkr} \)

Velocity field \( u \):

\[ \bar{u}(r) = -\frac{\nabla p}{j\omega \rho_o} = \hat{r} \frac{A}{\eta_s r} \left(1 + \frac{1}{jk r} \right) e^{-jkr} \]

Far-Field: \( kr >> 1 \quad \Rightarrow \quad r >> \lambda/2\pi \):

\[ p(r) = (A/r)e^{-jkr} \]

\[ \bar{u}(r) = (A/r\eta_s)e^{-jkr} = p(r)/\eta_s \]

Near-Field Radiation: Since \( \nabla \bar{p} = -j\omega \bar{u} \) therefore:

\[ p(r) = (A/r)e^{-jkr} \]

\[ \bar{u}(r) = (-jA/r^2k\eta_s)e^{-jkr} = -jp(r)/\rho_o \omega r \]

"Velocity mikes" close to the lips boost lows; need \( \times \omega \) compensation

* \[ \nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \]
ACOUSTIC ANTENNAS (2)

Antenna Gain \( G(\theta,\phi) \), Effective Area \( A(\theta,\phi) \) [m\(^2\)]:

\[
G(\theta,\phi) = \frac{P_r(\theta,\phi)}{(P_t / 4\pi r^2)}
\]

\[
P_{\text{received}} = I(\theta,\phi) A(\theta,\phi) \text{ [W]}
\]

Antenna (Loudspeaker, Microphone) Configurations:

- Monopole
- Dipole
- Baffled monopole
- Lense
- Parabolic dish
- Horn
- Array (end-fire or broadside)
**ACOUSTIC RESONATORS**

**A\textsubscript{mnp} Resonances of a Box:**

\[ m(\frac{\lambda_y}{2}) = a \implies \lambda_y = \frac{2a}{m} \]

\[ n(\frac{\lambda_x}{2}) = b, \quad p(\frac{\lambda_z}{2}) = d \]

\[ (\nabla^2 + k^2)p = 0 \]

\[ \text{e.g., } p = p_0 e^{-jk_x x - jk_y y - jk_z z} \]

\[ k^2 = \frac{\omega^2}{c_s^2} = k_x^2 + k_y^2 + k_z^2 = \left(\frac{2\pi}{\lambda_x}\right)^2 + \left(\frac{2\pi}{\lambda_y}\right)^2 + \left(\frac{2\pi}{\lambda_z}\right)^2 \]

**Resonant Frequencies of the A\textsubscript{mnp} Mode in a Box:**

\[ f_{mnp}^2 = \left(\frac{\omega}{2\pi}\right)^2 = \frac{c_s^2}{\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} + \frac{1}{\lambda_z^2}} \text{, where } \lambda_x = \frac{2b}{n} \]

\[ f_{mnp} = c_s \sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 + \left(\frac{p}{2d}\right)^2} \text{ [Hz]} \]

\[ f_{000} = 0 \text{ Hz (constant pressure)} \]

\[ f_{001} = \frac{340}{2d} \implies 170 \text{ Hz for a one-meter closed pipe} \]
Modal Density in Rectangular Resonators:

Recall: \( f_{mnp} = c_s \sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 + \left(\frac{p}{2d}\right)^2} \) [Hz]

Each cube has volume = \( c_s^3/8V \)
where \( V = abd \) (volume of resonator)

Number of modes in \( \Delta f \) ≅
(Volume of shell)/(vol. of cell) ≅ 
\( 4\pi f^2 \Delta f/[8(c_s^3/8V)] \) ≅ 
\( 4\pi f^2 \Delta f V/c_s^3 \) modes in \( \Delta f \)

Example:

Bathroom 3×3×3 meters
\( \Rightarrow \) lowest \( f_{100} = c_s/2a \approx 340/6 \approx 57 \) Hz

Modal density at 500 Hz \( \approx 4\pi \times 500^2 \times 1 \times 3^3/340^3 \approx 2 \) modes/Hz

How can we select just one mode when we sing a single note?
EXCITATION OF RESONATORS

**TEM Resonators (with loss):**

\[
I(t) = I_0 \cos \omega_0 t \\
V(\delta, t) = V_0 \cos(\omega_0 t + \phi) \sin(2\pi\delta/d) \\
\phi = 0 \text{ exactly at resonance} \\
P_{in}(t) \approx I_0 V_0 \cos^2(\omega_0 t) \sin(2\pi\delta/d) \ [W] \\
= 0 \text{ at voltage nulls}
\]

Cannot excite TEM\(_m\) modes by driving current into voltage nulls! (Or by voltage sources in series at current nulls). \(P_{in}(t) = 0\) in both cases.

**Acoustic Resonators:**

\[
I [\text{Wm}^{-2}] = p\vec{u} \cdot \hat{n}
\]

Cannot excite acoustic modes with:

- velocity sources at pressure nulls \((p_k = 0)\), or
- pressure sources at velocity nulls \((v_k = 0)\)

**Bathroom Opera:**

Mouth \(\approx\) velocity source

Place mouth near a pressure maximum of desired mode
HUMAN ACOUSTIC RESONATORS

Human Vocal Tract:

\[ f_1 = \frac{c_s}{\lambda_1} = \frac{c_s}{4d} \]
\[ = \frac{340}{(4 \times 0.16)} \]
\[ = 531 \text{ Hz} \]

Higher Resonances:

\[ f_2 = 3f_1 = 1594 \text{ Hz} \]
\[ f_3 = 5f_1 = 2655 \text{ Hz} \]

Energy Densities at Location “ ”

At \( f_1 \): \( w_p \approx w_u \)
At \( f_2 \): \( w_u >> w_p \)
At \( f_3 \): \( w_p << w_u \)
RESONANCE SHIFTS IN HUMAN VOICES

Human Vocal Tract:

Average force exerted by waves:
- Outward at maximum $|p|$ (max $w_p$)
- Inward at maximum $|u|$ (max $w_u$)
  (Bernoulli force)

Resonator Total Energy $w_T = nhf_o$:
- Pressing inward at $p_{\text{max}}$ increases $w_T$ and $f_o$
  (Phonon number $n = \text{constant for slow changes}$)
- Recall: pressure $\{N \text{ m}^{-2}\} \propto$ energy density $[J \text{ m}^{-3}]$

Resonance Perturbations:

$$\frac{\Delta f}{f} = \frac{\Delta(w_p - w_u)}{w_T}$$

- $w_p \gg w_u$ at $f_3$
- $w_u \gg w_p$ at $f_2$
- $w_p \cong w_u$ at $f_1$

Tongue position determines vowel

$vowels$