Problem Wk.3.3.3: Finding systems

A difference equation is in the form:

\[ y[n] = c_0 y[n-1] + c_1 y[n-2] + \ldots + c_k y[n-k] + d_0 x[n] + d_1 x[n-1] + \ldots + d_j x[n-j] \]

Determine the difference equation representation for the following systems.

Specify the \( d\text{Coeffs} \): \( d_0 \ldots d_j \) and the \( c\text{Coeffs} \): \( c_0 \ldots c_{k-1} \) for each of the difference equations below. For each question, enter a sequence of numbers representing the coefficients.

If one set of coefficients is empty, enter \text{none}, otherwise enter a sequence of numbers separated by spaces (no commas, parens, brackets, etc).

1. Let \( x[n] \) be an input sequence of digitized sound. We want to output a sound sequence \( y[n] \) where every output sample is the average of the previous two input samples, that is, \( n-1, \, n-2 \). Don't worry about what happens on the first few samples.

\[ \text{Difference equation:} \]
\[ d\text{Coeffs (input)}: \quad \]
\[ c\text{Coeffs (output)}: \quad \]

2. Assume that the input to a system is 0 for \( n < 0 \) and 1 for \( n \geq 0 \). The output is 0 for \( n < 0 \) and is equal to the sequence 10, 1, 1, 1, 1, \ldots \ for \( n \geq 0 \). Hint: the answer has no \( c\text{Coeffs} \).

\[ \text{Difference equation:} \]
\[ d\text{Coeffs (input)}: \quad \]
\[ c\text{Coeffs (output)}: \quad \]

3. A Bank offers a 5% annual interest rate, the inputs are your deposits, and the output is the balance in your account. Let \( x[n] \) represent the amount of money you deposit in the bank during year \( n \) and \( y[n] \) represent your balance in the bank at the end of year \( n \). Assume that deposits during year \( n \) are credited to the balance in year \( n \) but earn no interest until year \( n + 1 \).

\[ \text{Difference equation:} \]
\[ d\text{Coeffs (input)}: \quad \]
\[ c\text{Coeffs (output)}: \quad \]

4. Assume that you deposit $100 in the Bank in the year 2007 (\( n=0 \)) and make no further deposits. Solve your difference equation numerically to determine your balance in the bank during years 0 through 25. Hint: All this requires is a very simple Python program.

What is your bank balance in the year 2011 (\( n=4 \))? 
What is your bank balance in the year 2031 (\( n=24 \))? 
