The Circuit Abstraction

Circuits represent systems as connections of elements
- through which currents (through variables) flow and
- across which voltages (across variables) develop.

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Circuits represent the flashlight as a voltage source (battery) connected to a resistor (light bulb).

The voltage source generates a voltage \( v \) across the resistor and a current \( i \) through the resistor.

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Circuits are important for two very different reasons:
- as physical systems
  - power (from generators and transformers to power lines)
  - electronics (from cell phones to computers)
- as models of complex systems
  - neurons
  - brain
  - cardiovascular system
  - hearing

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Circuits are basis of enormously successful semiconductor industry.

What design principles enable development of such complex systems?
The Circuit Abstraction

Circuits represent systems as connections of elements
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Analyzing Circuits: Elements

We will start with the simplest elements: resistors and sources

\[ v = iR \]
\[ v = V_0 \]
\[ i = -I_0 \]

Analyzing Simple Circuits

Analyzing simple circuits is straightforward.

Example 1:

The voltage source determines the voltage across the resistor, \( v = 1V \), so the current through the resistor is \( i = v/R = 1/1 = 1A \).

Example 2:

The current source determines the current through the resistor, \( i = 1A \), so the voltage across the resistor is \( v = iR = 1 \times 1 = 1V \).

Check Yourself

What is the current through the resistor below?

1. 1A
2. 2A
3. 0A
4. cannot determine
5. none of the above

Analyzing More Complex Circuits

More complex circuits can be analyzed by systematically applying Kirchhoff’s voltage law (KVL) and Kirchhoff’s current law (KCL).
Analyzing Circuits: KVL

KVL: The sum of the voltages around any closed path is zero.

\[ v_1 = V_0 \]

Example: \(-v_1 + v_2 + v_4 = 0\) or equivalently \(v_1 = v_2 + v_4\).

How many other KVL relations are there?

Check Yourself

How many KVL equations can be written for this circuit?

1. 3 2. 4 3. 5 4. 6 5. 7

Analyzing Circuits: KVL

Planar circuits can be characterized by their “inner” loops.

KVL equations for the inner loops are independent.

A

B

C

KVL: Summary

The sum of the voltages around any closed path is zero.

One KVL equation can be written for every closed path in a circuit.

Sets of KVL equations are not necessarily linearly independent.

KCL equations for the “inner” loops of planar circuits are linearly independent.
Kirchhoff’s Current Law
The flow of electrical current is analogous to the flow of incompressible fluid (e.g., water).

Current $i_1$ flows into a node and two currents $i_2$ and $i_3$ flow out:

$i_1 = i_2 + i_3$

Kirchhoff’s Current Law
The net flow of electrical current into (or out of) a node is zero.

Here, there are two nodes, each indicated by a dot.

The net current out of the top node must be zero:

$i_1 + i_2 + i_3 = 0$.

Kirchhoff’s Current Law
Electrical currents cannot accumulate in elements, so current that flows into a circuit element must also flow out.

$i_1 = i_4$

$i_2 = i_5$

$i_3 = i_6$

Since $i_1 + i_2 + i_3 = 0$ it follows that

$i_4 + i_5 + i_6 = 0$.

Check Yourself
How many linearly independent KCL equations can be written for the following circuit?

Check Yourself
How many distinct KCL relations can be written for this circuit?

Analyzing Circuits: KCL
The net current out of any closed surface (which can contain multiple nodes) is zero.

node 1: $i_1 + i_2 + i_3 = 0$

node 2: $-i_2 + i_4 + i_6 = 0$

nodes 1+2: $i_1 + i_2 + i_3 - i_2 + i_4 + i_6 = i_1 + i_3 + i_4 + i_6 = 0$
**Analyzing Circuits: KCL**

The net current out of any closed surface (which can contain multiple nodes) is zero.

\[
\begin{align*}
\text{nodes 1+2:} & \quad i_1 + i_3 + i_4 + i_6 = 0 \\
\text{node 3:} & \quad -i_4 - i_6 + i_5 = 0 \\
\text{nodes 1+2+3:} & \quad i_1 + i_3 + i_4 - i_3 - i_6 + i_5 = i_1 + i_4 + i_5 = 0
\end{align*}
\]

Net current out of nodes 1+2+3 = net current into bottom node!

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**Node Voltages**

The “node” method is one (of many) ways to systematically reduce the number of circuit equations and unknowns.

- label all nodes except one: ground (gnd) \(\equiv 0\) volts
- write KCL for each node whose voltage is not known
- solve (here just 2 equations and 2 unknowns)

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**KVL, KCL, and Constitutive Equations**

Circuits can be analyzed by combining

- all linearly independent KVL equations,
- all linearly independent KCL equations, and
- one constitutive equation for each element.

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**KCL: Summary**

The sum of the currents out of any node is zero.

One KCL equation can be written for every closed surface (which contain one or more nodes) in a circuit.

Sets of KCL equations are not necessarily linearly independent.

KCL equations for every primitive node except one (ground) are linearly independent.

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**Loop Currents**

The “loop current” method is another way to systematically reduce the number of circuit equations and unknowns.

- label all the loop currents
- write KVL for each loop
- solve (here just 3 equations and 3 unknowns)
Analyzing Circuits: Summary

We have seen three (of many) methods for analyzing circuits. Each one is based on a different set of variables:
- currents and voltages for each element
- node voltages
- loop currents
Each requires the use of all constitutive equations.
Each provides a systematic way of identifying the required set of KVL and/or KCL equations.

Check Yourself

Determine the current $I$ in the circuit below.

1. 1 A 2. $\frac{1}{2}$ A 3. $-1$ A 4. $-5$ A 5. none of the above

Common Patterns

Circuits can be simplified when two or more elements behave as a single element.
A “one-port” is a circuit that can be represented as a single element.

A one-port has two terminals. Current enters one terminal (+) and exits the other (−), producing a voltage ($v$) across the terminals.

Series Combinations

The series combination of two resistors is equivalent to a single resistor whose resistance is the sum of the two original resistances.

$$v = R_1 i + R_2 i \quad v = R_s i$$
$$R_s = R_1 + R_2$$
The resistance of a series combination is always larger than either of the original resistances.

Check Yourself

What is the equivalent resistance of the following one-port.

1. 0.5 2. 1 3. 2 4. 3 5. 5

Parallel Combinations

The parallel combination of two resistors is equivalent to a single resistor whose conductance (1/resistance) is the sum of the two original conductances.

$$i = \frac{v}{R_1} + \frac{v}{R_2} \quad i = \frac{v}{R_p}$$
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \quad R_p = \frac{R_1 R_2}{R_1 + R_2} = R_1 || R_2$$
The resistance of a parallel combination is always smaller than either of the original resistances.
Voltage Divider
Resistors in series act as voltage dividers.

\[ I = \frac{V}{R_1 + R_2} \]
\[ V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V \]
\[ V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V \]

Current Divider
Resistors in parallel act as current dividers.

\[ V = \left( R_1 || R_2 \right) I \]
\[ I_1 = \frac{V}{R_1} = \frac{R_1 || R_2}{R_1} I = \frac{R_1 R_2}{R_1 + R_2} I = \frac{R_2}{R_1 + R_2} I \]
\[ I_2 = \frac{V}{R_2} = \frac{R_1 || R_2}{R_2} I = \frac{R_1 R_2}{R_2 + R_1} I = \frac{R_1}{R_1 + R_2} I \]

Check Yourself
15V
1Ω 3Ω
12Ω 6Ω Vo
Which of the following is true?
1. \( V_0 \leq 3V \)
2. \( 3V < V_0 \leq 6V \)
3. \( 6V < V_0 \leq 9V \)
4. \( 9V < V_0 \leq 12V \)
5. \( V_0 > 12V \)

Summary
Circuits represent systems as connections of elements
- through which currents (through variables) flow and
- across which voltages (across variables) develop.

We have seen three (of many) methods for analyzing circuits. Each one is based on a different set of variables:
- currents and voltages for each element
- node voltages
- loop currents

We can simplify analysis by recognizing common patterns:
- series and parallel combinations
- voltage and current dividers