Problem Wk.10.1.2: Conditional distributions

Read Section 7.2 of the course notes if you haven't already done so. It is important to be able to represent conditional distributions, of the form \( P(B \mid A) \). Conditional distributions are not, themselves, distributions. We will represent conditional distributions as functions from values that the variable \( A \) can take on, to distributions over \( B \).

So, if \( \text{bar} \) is a conditional distribution representing \( P(B \mid A) \), then \( \text{bar}(a) \) will be the distribution over \( B \) corresponding to \( P(B \mid A = a) \).

For example, consider a situation where we have the variable \( \text{Year} \) with domain \( (1,2,3,4) \) and the variable \( \text{Grade} \) with domain \( ('a', 'b', 'c', 'd', 'f') \). We can represent the conditional distribution \( P(\text{Grade} \mid \text{Year}) \) as a function, called \( \text{PGgY} \), which, given a value for \( \text{Year} \), returns a \( \text{DDist} \) over the grades:

```python
>>> PGgY(1)
DDist(a: 0.3, b: 0.3, c: 0.3, d: 0.07, f: 0.03)
>>> PGgY(2)
DDist(a: 0.25, b: 0.35, c: 0.3, d: 0.07, f: 0.03)
```

Let variable \( \text{Disease} \) have domain \( ('disease', 'noDisease') \) and let variable \( \text{Test} \) have domain \( ('posTest', 'negTest') \).

Define a conditional distribution (a function) that takes a value of \( \text{Disease} \) and returns a \( \text{DDist} \) that represents the distribution of \( \text{Test} \), conditioned on \( \text{Disease} \) having the specified value. The function should encode the facts that:

- \( P(\text{posTest} \mid \text{disease}) = 0.98 \), and that
- \( P(\text{posTest} \mid \text{noDisease}) = 0.05 \).

Think carefully about the distributions that are consistent with these facts.

For example: \( \text{PTgD('disease').prob('posTest')} \) should evaluate to 0.98.

This is happening in the \text{dist} module, so you do not need to use \text{dist.DDist}, it's enough to type \( \text{DDist} \).