1. See the textbook, Problem 2.35, page 130.

2. (a) \[ p_X(1) = P(X = 1, Y = 1) + P(X = 1, Y = 2) + P(X = 1, Y = 3) = \frac{1}{12} + \frac{2}{12} + \frac{1}{12} = \frac{1}{3} \]

(b) The solution is a sketch of the following conditional PMF:

\[
\begin{align*}
p_{Y \mid X}(y \mid 1) &= \frac{p_{Y, X}(y, 1)}{p_X(1)} = \begin{cases} 
1/4, & \text{if } y = 1, \\
1/2, & \text{if } y = 2, \\
1/4, & \text{if } y = 3, \\
0, & \text{otherwise.}
\end{cases}
\end{align*}
\]

(c) \[ E[Y \mid X = 1] = \sum_{y=1}^{3} y p_{Y \mid X}(y \mid 1) = 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4} = 2 \]

(d) Assume that \( X \) and \( Y \) are independent. Because \( p_{X,Y}(3, 1) = 0 \) and \( p_{Y}(1) = 1/4, p_{X}(3) \) must equal zero. This further implies \( p_{X,Y}(3, 2) = 0 \) and \( p_{X,Y}(3, 3) = 0 \). All the remaining probability mass must go to \((X, Y) = (2, 2)\), making \( p_{X,Y}(2, 2) = 5/12, p_{X}(2) = 8/12, \) and \( p_{Y}(2) = 7/12 \). However, \( p_{X,Y}(2, 2) \neq p_{X}(2) \cdot p_{Y}(2) \), contradicting the assumption; thus \( X \) and \( Y \) are not independent.

A simpler explanation uses only two \( X \) values and two \( Y \) values for which all four \((X, Y)\) pairs have specified probabilities. Note that if \( X \) and \( Y \) are independent, then \( p_{X,Y}(1, 3)/p_{X,Y}(1, 1) \) and \( p_{X,Y}(2, 3)/p_{X,Y}(2, 1) \) must be equal because they must both equal \( p_{Y}(3)/p_{Y}(1) \). This necessary equality does not hold, so \( X \) and \( Y \) are not independent.

(e) Knowing that \( X \) and \( Y \) are conditionally independent given \( B \), we must have

\[
\frac{p_{X,Y}(1, 1)}{p_{X,Y}(1, 2)} = \frac{p_{X,Y}(2, 1)}{p_{X,Y}(2, 2)}
\]

since the \((X, Y)\) pairs in the equality are all in \( B \). Thus

\[
p_{X,Y}(2, 2) = \frac{p_{X,Y}(1, 2)p_{X,Y}(2, 1)}{p_{X,Y}(1, 1)} = \frac{(2/12)(2/12)}{1/12} = \frac{4}{12} = \frac{1}{3}.
\]

(f) Since \( P(B) = 9/12 = 3/4 \), we normalize to obtain \( p_{X,Y \mid B}(2, 2) = \frac{p_{X,Y}(2, 2)}{P(B)} = 4/9 \).

3. See the textbook, Problem 2.33, page 128.